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SIMULATION OF THE COMPATIBILITY OF AN AIR  
CAPABLE SHIP AND A VTOL AIRCRAFT

George H. Daffer, et al

CADCOM, Incorporated

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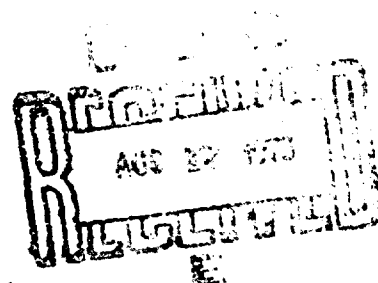
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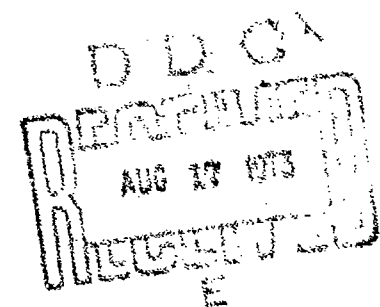


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AN AIR CAPABLE SHIP AND A VTOL AIRCRAFT  
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George H. Daffer and David F. Rogers



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by George H. Daffer and David F. Rogers

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18. ABSTRACT An interactive computer simulation, LARC-I, has been designed to solve the non-linear equations of motion of a generalized VTOL aircraft taking off from or landing on the deck of a ship moving in an irregular or random seaway. This version of LARC-I is limited to longitudinal motions, but is designed for eventual expansion to all degrees of freedom. The LARC-I programs makes use of ship motion amplitudes and frequencies derived separately in a ship motions program, wherein the forcing functions of the seaway are based on a stochastic representation of the waves for any given sea state. The pitching and heaving motions of the ship are transmitted to the aircraft by a realistic simulation of the landing gear. The program permits solution of an arbitrary maneuver, defined by a sequence of aircraft control changes, and composed of any number of segments of arbitrary length, in each of which the controls remain fixed. The equations of motion for a generalized VTOL aircraft are particularized by adding a force and moment module for each specific aircraft. This version of LARC-I contains one such module for the H-53A, D helicopters. Dynamic validation with flight test data is considered excellent.	

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## SUMMARY

### Project Description

This report describes the work accomplished by CADCOM, Inc., under OAR Contract N00014-72-C-0531, titled "Sea Control Ship/Aircraft Motions". The work was sponsored jointly by the Office of Naval Research and the Naval Air Systems Command.

The purpose of the project was to develop an analytical, computer-augmented, simulation to provide: (a) a launch/recovery effectiveness evaluation of ship-aircraft combinations under a variety of wind/wave and ship motion conditions, and (b) a specific effectiveness evaluation directed to a Sea Control Ship and the VTOL aircraft to be associated with its operation. The principal objective was the construction and subsequent evaluation of an accurate and representative simulation model.

It was established that certain limiting assumptions and simplifications would be necessary in order to make the development of a simulation possible under the given funding and time constraints and that the exercise of the model would be necessarily confined to two candidate sea control ships and two aircraft (CH-53 and Harrier).

The simulation package which was developed under the contract is called "LARC-I" (Launch and Recover CompatibIlity). LARC-I simulates the motion of an aircraft in an irregular seaway. Although the motions of a aircraft are confined to the x-z plane and those of the ship to heave, pitch and a forward motion, the LARC-I program is

structured to allow an orderly modular expansion into a version which will simulate the motion in all six degrees of freedom, longitudinal and lateral.

The ship motions selected for the exercise of the model are parametric in nature and hence can represent the motions of a number of candidate air-capable ships. The aircraft used in the model exercise and generation of data was the CH-53D helicopter. During the performance period, ADCOM, ONR, and NAVAIR attempted to obtain data on the Harrier aircraft without success. Thus, the latter aircraft was eliminated from the computational phase of the project and, instead, additional data was generated for the CH-53D.

### Conclusions

Data on the Harrier aircraft was received by ADCOM after the completion of this project, and it is included in this report (Appendix E) for reference purposes.

As a result of this work, the tentative conclusion has been reached that pitching and heaving platform motions will not limit normal take-off and recovery maneuvers of a VTOL aircraft of this type. Natural frequencies are too far apart to couple. Although there is an increase in control frequencies due to fuselage pitching moments, the required pilot response is within normal limits. Specifically:

- a. For the ship motion conditions investigated, no serious stability problem occurred which would limit the take-off capability of the CH-53D helicopter. The static



and dynamic stability margins of the aircraft are sufficient to allow safe take-off maneuvers with acceptable levels of pilot effort.

b. In no case were landing gear loads in excess of the design limit loads as given in References 6 and 7. (Note)

#### Recommendations

As a result of its experience with surveying the state of the art, developing the mathematical model, constructing the computer simulation, and applying LARC-I to realistic ship/aircraft operations, CADCOM makes the following recommendations:

a. Successful and realistic general use of the LARC simulation as an analytical tool in system analysis will require the extension of the program to all six degrees of freedom for both ship and aircraft. It is apparent from the results of the longitudinal motion studies that high-frequency cyclic response will result in roll forces which may couple with landing gear forces in the roll plane.

b. Real-time or quasi-real-time flight control should be added to the simulation to allow the operator to "fly" the model. His reference system would be provided by an interactive computer graphics display. In the present version, the model operates in short (5-8 second) control-fixed segments. In the proposed interactively controlled model, the

Note: Design limit loads - main gear: 21500<sup>l</sup>  
(each gear - cond. "L. L.")  
nose gear: 17100<sup>l</sup>  
(each gear - cond. L. L. 3 PT.)

high sampling rate control loop would allow adaptive control in a real-time mode which more accurately reflects pilot response.

c. Investigation of the mathematical representation of the random sea with the intention of resolving the problems discussed in Appendix D should be continued.

## I. INTRODUCTION

Frequently the effect of the motion of a ship in moderate to heavy seas on the aircraft and/or the ship design and performance characteristics necessary for a high probability for a successful launch or recovery of aircraft have been given little consideration during the design phase. In fact, the design trade-offs between the ship and the aircraft performance characteristics are probably unknown to a large degree.

Recently the Sea Control Ship has emerged as a significant advancement in our defense posture. As presently conceived, the Sea Control Ship is a small, versatile combatant with significant air capability. If this new concept is to succeed, significant effort must be expended to insure that small ships can serve as adequate platforms for aircraft. Since the platforms on such ships will of necessity be smaller, the motions of those platforms will consequently be more severe than those of present aircraft carriers.

One of the first steps in the process of evaluating the air capable potential of candidate ship types and/or designs is to determine their seakeeping characteristics. Once the response of the candidate ship to various sea states is known, this information can be assessed together with the maneuverability and control characteristics of candidate aircraft to determine the potential as a Sea Control Ship or aircraft.

Using computer-aided analysis and interactive graphics, techniques it is possible to simulate the individual motions of both the aircraft and the ship and hence their relative motions. This leads to the ability to estimate the probability of a successful launch/recovery with existing aircraft from existing or proposed ships. In addition, it is possible to simulate the motion of proposed ship concepts and thus to determine the probability of a successful launch/recovery with existing or proposed aircraft.

In the following discussion, the motion of the ship is first discussed in Section II, then the motion of the aircraft in Section III, and finally the combined motion as used in the simulation as well as the simulation itself in Section IV.

## II. SHIP MOTIONS

The characteristics of the ship which were used in this simulation are shown in Table II-1. Further details are given in detail in References 1 and 2. The determination of the ship response amplitude operator (RAO) was by means of the linear superposition theory described in References 1 through 3 except for the pitch RAO at a Froude number of 0.30. These responses were derived from model tests. The theoretical and experimental values are shown in Figure II-1.

TABLE II-1

MODEL M

PARAMETER	SHIP MODEL VALUES
LENGTH BETWEEN PERPENDICULARS LBP, FT	12.69
DRAFT AT $\overline{II}$ , FT	0.715
BLOCK COEFFICIENT	0.632
BEAM, FT	1.83
CENTER OF BUOYANCY, FT. AFT OF $\overline{II}$ STATIONS	0.208 10.33
CENTER OF FLOTATION, FT AFT OF $\overline{II}$ STATIONS	.524 10.83
RADIUS OF GYRATION/LBP, FT	0.256

The full scale length of the ship was assumed to be 630' LWL and LBP. The maximum speed of the vessel was assumed to be about 25 knots, corresponding to a Froude number

$$Fr = \frac{U}{\sqrt{gL}} = 0.30$$

II-1

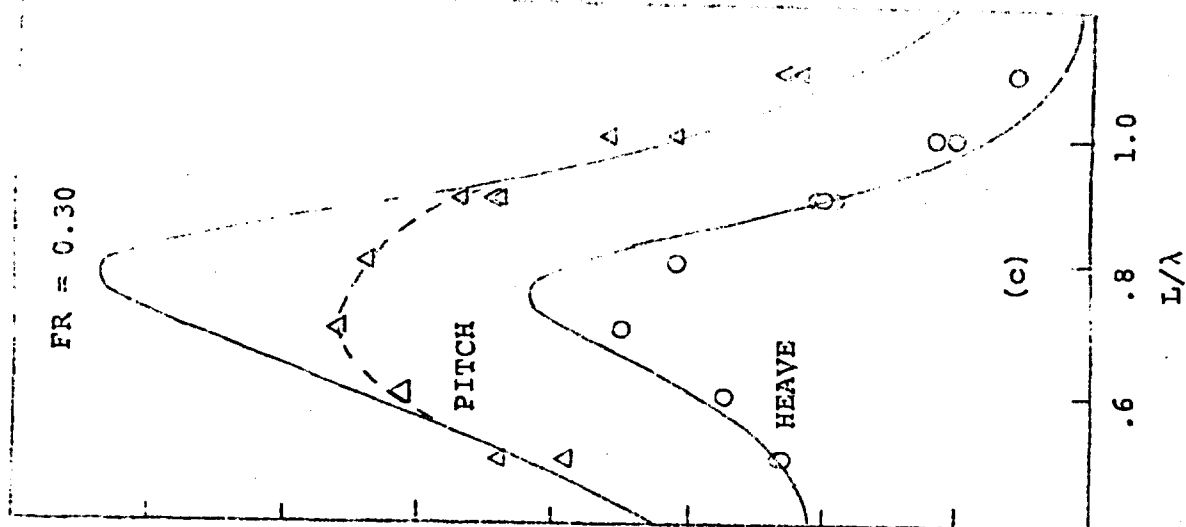
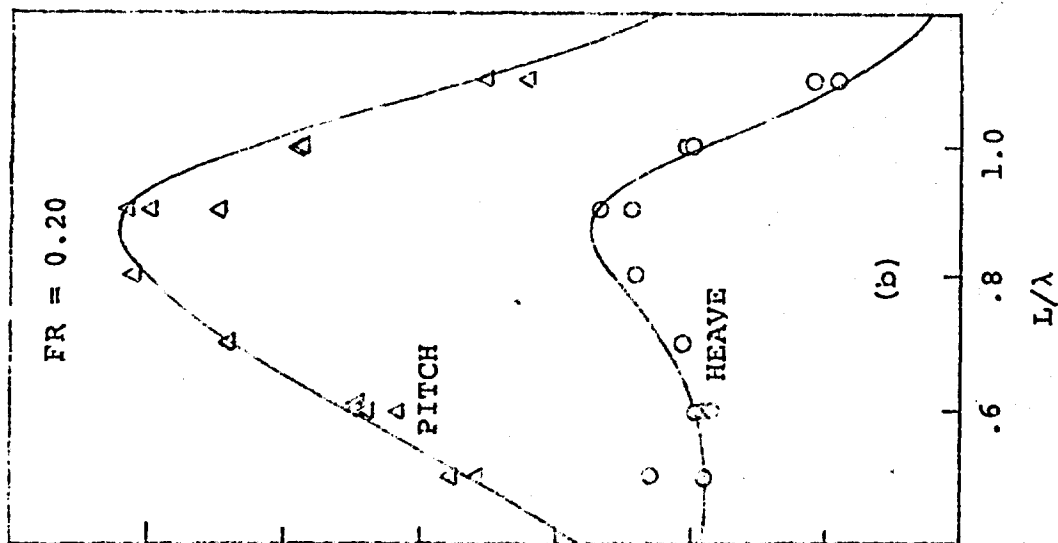
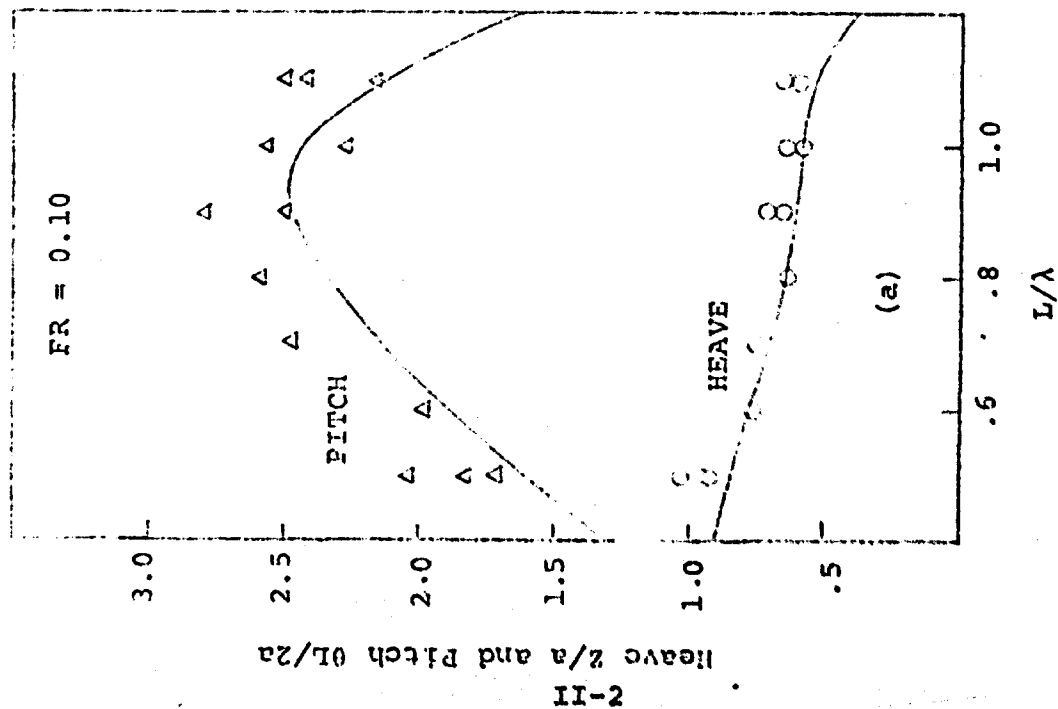


Figure II-1 Experimental and Theoretical Heave and Pitch Amplitudes for Model M Hull at FR. = 0.10, 0.20 and 0.30

The data shown in Figure II-1 are for Froude numbers of 0.10, 0.20 and 0.30, and for wave lengths ranging from  $L/\lambda = 0.50$  to 1.10. In only one case were the experimental results substituted for the theoretical curve; the pitch response at  $Fr = 0.30$ . Except for this case, theory and experiment are in close agreement. At  $Fr = 0.30$ , the theory overestimates the pitch response by up to 38 percent. The authors of Ref. 1 attribute this to large "steady state" waves which may be expected at and above the design hull speed. The design speed for the Model M hull is about  $Fr = 0.28$ , or for a 630' ship, about 23.5 knots. Unless the interaction between the oscillatory motion and "steady state" Kelvin wave pattern is accounted for in the theory, strip theory may be expected to overestimate amplitudes in the speed regime where such waves affect the pitching response. For pitching amplitudes at  $Fr = 0.30$  the dashed curve of Figure III-1c was used. This curve was coincident with the theoretical curve between  $L/\lambda = 0.9$  to 1.2.

The Pierson-Moskowitz sea spectrum was used as the representation of the random seas. This spectrum is

$$S_a(\omega) = \frac{a \omega^{-5}}{e^{\frac{b}{\omega^4}}} \quad \text{II-2}$$

where  $a = 0.0081$

$b = 11.56$

$h_{1/3}$  = significant wave height, ft.

$\omega$  = wave frequency, radians per second

$g$  = acceleration of gravity, ft/sec<sup>2</sup>

The spectral values for significant wave heights of 15, 20, 25 and 30 ft are shown in Figure II-2. This formula gives an approximation to real sea conditions which has been recommended by the 11th International Towing Tank Conference (1966) for use "when information on typical sea spectra is not available." The shapes of spectral curves derived from data collected in different sea locations and at different periods of the year show wide variations. In future studies, it is recommended that more representative data be used. At this stage (in this project) the Pierson-Moskowitz spectrum is adequate since the principal objective of the investigations described herein is the demonstration of the simulation model.

The curves of Figure II-2 approximate the spectra which would be derived from data collected at a stationary point. If the point were moving at a steady velocity in a straight line, the curves would be displaced to the right as shown in Figure II-3. The displaced curve in Figure II-3 is the 20-foot spectrum as it would appear to a ship moving at 25.3 knots or 42.8 FPS. To map the spectral data from the stationary  $\omega - \chi$  plane (where  $\omega$  is wave frequency and  $\chi$  is wave heading) into the moving  $\omega_e - \chi_e$  plane, where  $\omega_e$  is the frequency of encounter, where  $\omega_e$  and  $\omega$  are related by

$$\omega_e = \omega \left( 1 + \frac{u}{g} \cos \chi \right) \quad \text{II-3}$$



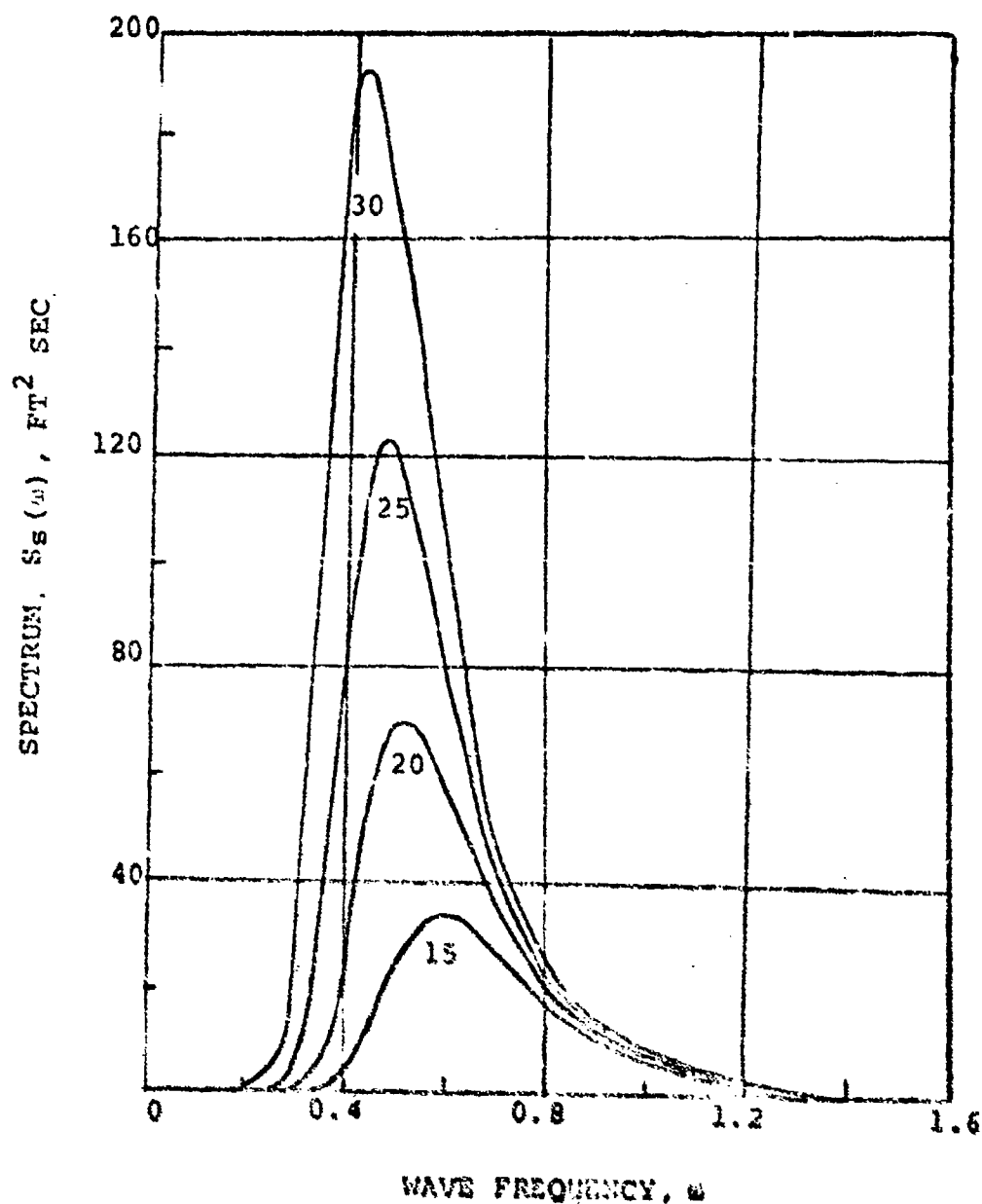


Figure II-2 The Pierson-Moskowitz Sea Spectrum for 15-, 20-, 25-, and 30-Foot Significant Wave Heights

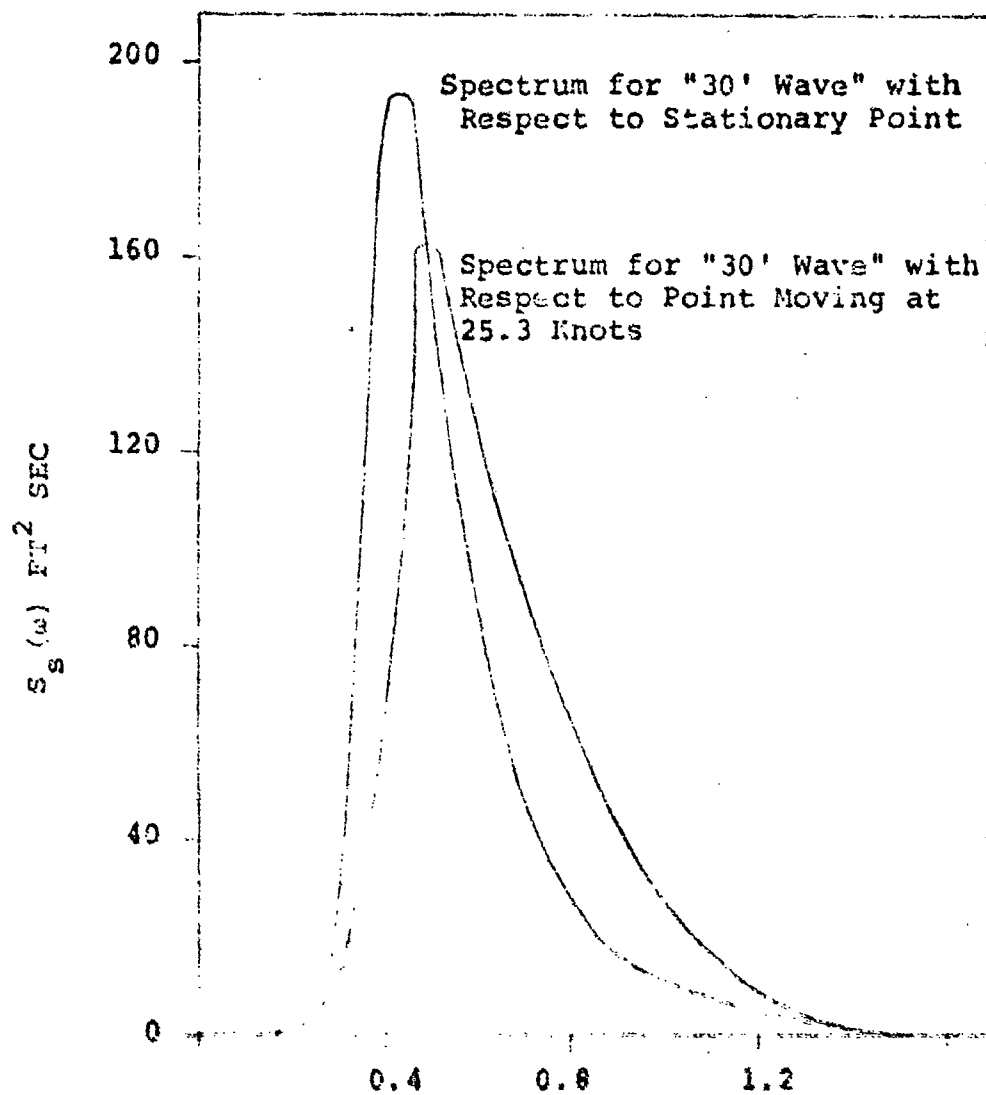


Figure 11-3 Sea Spectra for Stationary and Moving Point - Significant Wave Height = 30 Ft.

it is necessary to make use of the Jacobean of the transformation

$$J \frac{\omega, \chi}{\omega_e, \chi_e} \triangleq \frac{\partial(\omega, \chi)}{\partial(\omega_e, \chi_e)} \triangleq \begin{vmatrix} \frac{\partial \omega}{\partial \omega_e} & \frac{\partial \omega}{\partial \chi_e} \\ \frac{\partial \chi}{\partial \omega_e} & \frac{\partial \chi}{\partial \chi_e} \end{vmatrix} \quad \text{II-4}$$

For the head-on condition, where the ship is overtaking the waves and traveling in the same direction, the Jacobean is

$$\frac{1}{(1 + \frac{4\omega_e}{g} U)^{1/2}} \quad \text{II-5}$$

Each ordinate of the "stationary" curve is multiplied by the above expression to give the corresponding ordinate of the transformed spectra, as it would appear to a moving ship (Reference 3).

Typical values of wave frequency, encounter frequencies and wave lengths for the 630' ship at  $Fr = 0.30$  are shown in Table II-2 below.

TABLE II-2

$L/\omega$	$\lambda$	$\omega_e$	$\omega$
.4	1575	.358	.265
.8	767	.507	.356
1.3	525	.621	.405

The wave length  $\lambda$  and the wave frequency  $\omega$  are related by the equation

$$\omega_e = \left[ 5.118 \left( \frac{4\pi^2}{\lambda} \right) \right]^{1/2} \approx \frac{14.22}{\sqrt{\lambda}} \quad \text{II-6}$$

For  $U = 42.3$  fps,  $\omega$  is given by

$$\omega \approx \frac{3}{8} \pm 1/2 \sqrt{\frac{9}{16} + 3\omega_e} \quad \text{II-7}$$

The transformed sea spectra and the ship's response amplitude operator squared are now multiplied ordinate-by-ordinate to obtain the response amplitude spectra for a given sea state and ship velocity. Examples are given in Figure II-4 for significant wave heights of 15 - 20 ft (sea state 6) and 25 - 30 ft (sea state 7), for the ship Model M moving at  $Fr = 0.30$ . Thus, the response energy spectrum as a function of encounter frequency  $\omega_e$  is

$$S_R(\omega_e) = [R(\omega_e)]^2 \cdot S_S(\omega_e) \quad \text{II-8}$$

The total energy of the response spectrum is by definition

$$E_R = \int_0^\infty S_R(\omega_e) d\omega_e \quad \text{II-9}$$

and assuming a Rayleigh distribution of the maxima ("amplitudes") the following response values may be expressed:

$$\text{the average "amplitude"} \quad a_{av} = 1.253 \sqrt{E_R} \quad \text{II-10}$$

$$\text{the significant "amplitude"} \quad a_{1/3} = 2.000 \sqrt{E_R} \\ \text{(average of the 1/3 highest)}$$

$$\text{the "1/10 highest"} \quad a_{1/10} = 2.546 \sqrt{E_R} \\ \text{(average of the 1/10 highest)}$$

For a Froude number of 0.30 the heave and pitch motions induced at the ship's c.g. by irregular seas of given significant wave heights are shown in Table II-3.

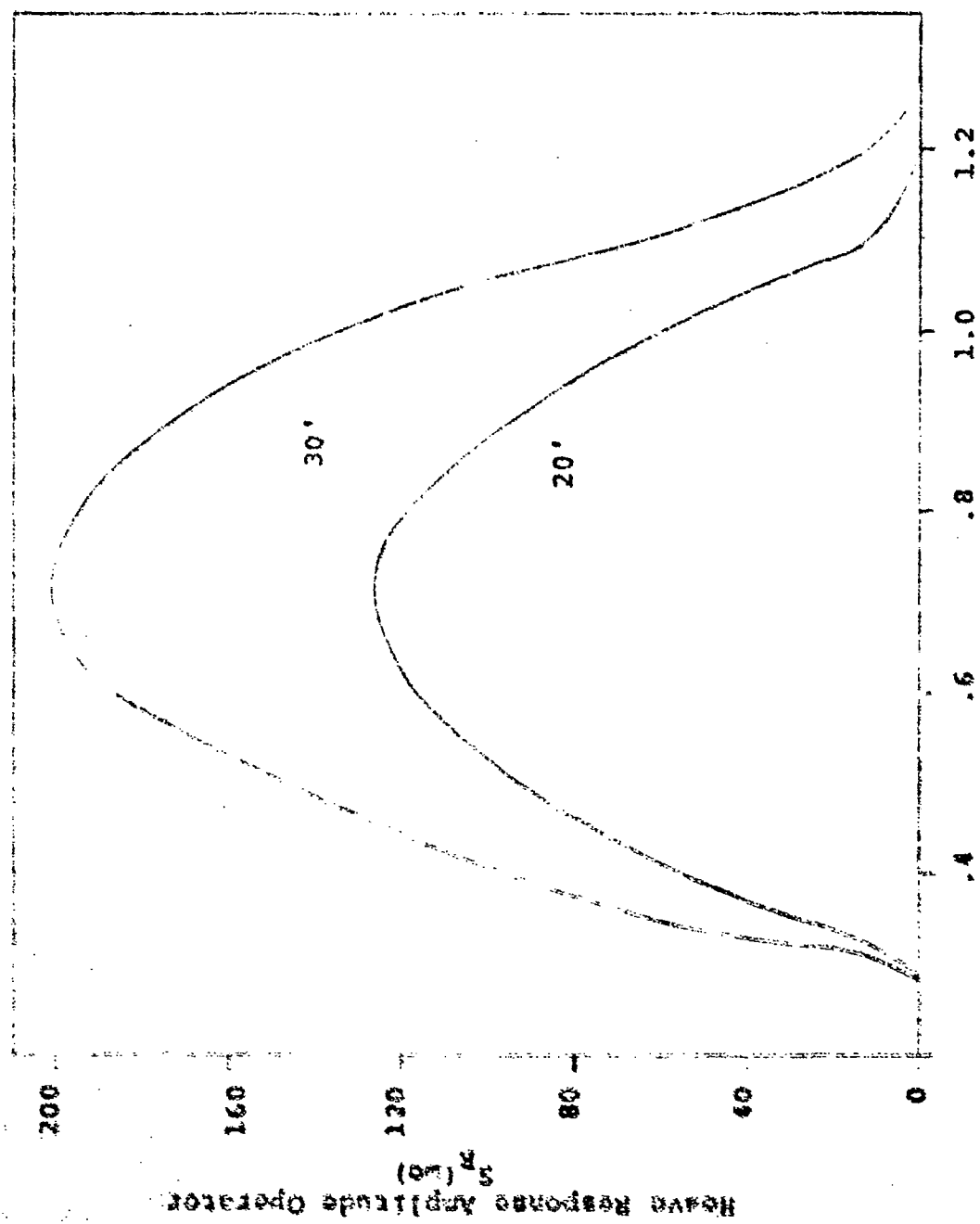


Figure 11-4 Heave Response Amplitude Operator  
for Model M Hull in Two Sea States

TABLE II-3  
SIGNIFICANT MOTIONS  
(Avg. of 1/3 Highest)

SIGNIFICANT WAVE HEIGHT (FT.)	HEAVE AMPLITUDE (FT.)	PITCH AMPLITUDE (DEGREES)
15	8.932	2.113
20	11.207	3.163
25	14.514	4.321
30	18.035	5.492

The amplitudes shown in Table II-3 are those used in the program to calculate motions at the landing location.

The Pierson-Moskowitz Spectrum which was used in this determination of heave and pitch amplitudes expresses the energy in fully developed seas and it depends on only one parameter, the significant wave height  $h_{1/3}$ . With more parameters one could represent the sea spectra actually encountered which are for the most part not fully developed. Or, more accurately, a spectrum could be used, derived from actual wave data at a given location. This would provide the highest measure of accuracy but would necessarily apply only to conditions which may occur at that geographic location at that time of year. The Pierson-Moskowitz spectrum is used in this study because it is simple to apply and gives reasonable predictions.

Ordinarily, the simulation would determine the heave amplitude of any point other than the center of gravity by evaluation of the heave response spectrum

$$[R_{z,x}(\omega_e)]^2 = [R_{z,cg}(\omega_e)]^2 + \ell^2 [R_{\theta,cg}(\omega_e)]^2 + 2\ell R_{z,cg}(\omega_e) R_{\theta,cg}(\omega_e) \cos(\delta(\omega_e) - \epsilon(\omega_e)) \quad \text{II-11}$$

where  $R_{z,x}$  is the response amplitude operator in heave (z) at any point x

$R_{z,cg}$  is the RAO in heave at the c.g.

$R_{\theta,cg}$  is the RAO in pitch at the c.g.

$\ell$  is the distance from the c.g. to the point x,

$\omega_e$  is the encounter frequency and

$\delta, \epsilon$  are the phase angles for pitch and heave respectively both measuring the lead of the ship response with respect to the maximum wave elevation at the midship location.

The ship vertical displacement spectrum at any point other than the c.g. due to the combined motions of heave and pitch are determined from this expression in the same fashion as the heave and pitch separately at the c.g. That is:

$$S_{R_{z,x}}(\omega_e) = [R_{z,x}(\omega_e)]^2 \cdot S_s(\omega_e) \quad \text{II-12}$$

and the significant response amplitude is

$$z_x(\omega_e) = 2.0 \left( \int_0^\infty S_{R_{z,x}}(\omega_e) d\omega_e \right)^{1/2} \quad \text{II-13}$$

where

$S_{R_{z,x}}$  is the RAO in the z direction (vertical) at the point x due to pitch and heave at the c.g.

$Z_x(\omega_e)$  is the significant vertical displacement at the point x.

Although the results derived from this procedure are statistically correct, they are difficult to use because the motions due to pitch and heave are coupled in the resulting amplitude  $Z_x(\omega_e)$ . Use of this amplitude alone precludes the analysis of the effects of heave and pitch motions at the point x separately. It would be preferable to be able to use an expression of the form

$$Z'_x(\omega_e) = Z_{cg} \cos(\omega_e t - \delta) + \ell \theta_{cg} \cos(\omega_e t - \epsilon) \quad \text{II-14}$$

where  $Z'_x$  is the vertical displacement at x

$Z_{cg}$  is the maximum vertical displacement to the cg

$\theta_{cg}$  is the maximum pitch displacement.

Or more conveniently

$$Z'_x(\omega) = Z_{cg} \cos(\omega'_e t) + \ell \theta_{cg} \cos(\omega'_e t - \delta')$$

$$\text{where } \omega'_e = \frac{1}{t} (\omega_e t - \delta)$$

II-15

$$\text{and } \delta' = (\delta + \epsilon)$$

$$\text{At } \omega_e t = 0 \quad \cos(\omega_e t) = 1$$

$$\sin(\omega_e t) = 0$$

$$\text{and } Z'_x = Z_{cg} + \ell \theta_{cg} \sin \delta'$$

II-16



If this is set equal to  $z_x(0)$ , the coupled pitch/heave deflection at  $x$ ,  $\cos \delta'$  and hence  $\delta'$  can be determined.

An example of a calculation is shown below to illustrate the use of this method.

The ship motions program predicts the following motions at stations 10 (approximate c.g.) and 5 (157' forward) for a 30' wave condition

	10	STATION	5
VERTICAL	18.03'		28.56'
PITCH	5.49°		5.49°

$$(\delta') \equiv \delta' \equiv \frac{28.53 - 18.03}{157 \times 0.0958} = 0.698 \text{ RAD. II-17}$$

At first this result appears to contradict the phase relations predicted by the ship motions program. Figure II-5 shows these relations for the  $L/\lambda$  range from 0.4 to 1.2 for the Model M. The heave and pitch motions are everywhere out of phase by between  $90^\circ$  and  $135^\circ$ . However since the pitch always lags the heave by this amount, the vertical motions are additive and thus Equation II-15 is a very good approximation of the maximum absolute deflection when  $\delta'$  is determined in this manner. The harmonic motion at the point  $x$  can be assumed very nearly equal to

$$\begin{aligned} z_x &= z'_x \cos(\omega_e t) \\ \theta_x &= \theta_{cg} \cos(\omega_e t) \end{aligned} \quad \text{II-18}$$

with  $z'_x$  determined by Eq. II-16 and with  $\delta'$  calculated as in Eq. II-17 for the particular point 157' ahead of the

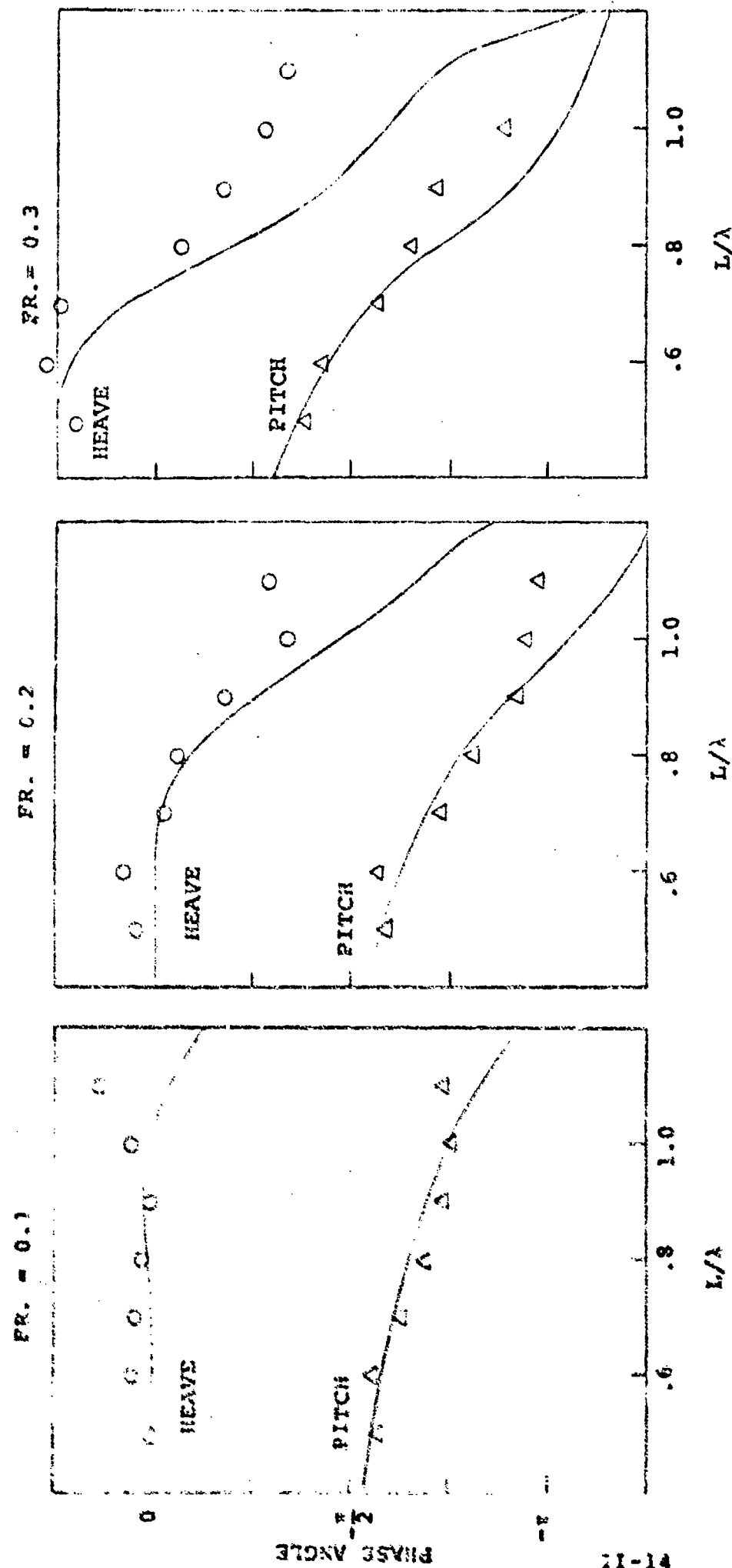


Figure II-5 Theoretical and Experimental Phase Angles for Model M

cg, for a sea state spectrum based on a significant wave height  $h_{1/3} = 30$  and a ship forward velocity of 25 knots.

The final problem, which must be solved in order to make the ship motions data useful in an investigation such as this, is the selection of a realistic value for  $\omega_e$ . The procedure which has been used is to set the value of the encounter frequency equal to the ratio of the integrals of the amplitude and velocity response spectra in heave. (The difference in this value and the corresponding value for the pitch spectra is always very small, never greater than 0.02 rad/sec.) In effect, this is the ratio of the total energy of the response spectrum and the first moment of that spectrum, or

$$\omega_{e \text{ sig}} = \frac{\int_0^{\infty} \omega_e S_R(\omega_e) d\omega_e}{\int_0^{\infty} S_R(\omega_e) d\omega_e} \quad \text{II-19}$$

(It makes no difference, incidentally, whether the sea spectrum  $S_g$  and the RAO are computed for wave frequencies,  $\omega$ , or encounter frequencies,  $\omega_e$ . That is to say the same values result when the sea spectrum  $S_g(\omega)$  and RAO,  $R(\omega)$ , are transformed separately into the encounter frequency plane, and then multiplied,  $(R(\omega_e))^2 \cdot S_g(\omega_e)$  to give the ship response spectrum  $S_R(\omega_e)$ , as when the response spectrum  $S_R(\omega)$  transforms into the same plane without prior shifting of the component curves.)

Eq. II-19 gives the value of a weighted average where the weights are the squared amplitudes of the ship response associated with that particular frequency. Since the  $S_R$  curve does not by nature possess multiple maxima, Eq. II-19 computes a weighted value associated with the wave components (wave lengths) to which the ship can respond and to wave heights in the sea spectrum,  $S_s(\omega_e)$ , which possess significant energy content to excite the ship. Since wave length and wave height are independent, this is the only feasible way of associating these values. It is important to note that  $\omega_e$  is not necessarily the most probable value. For example, if the ship response curve is flat through a range of frequencies, thus

$$\frac{dS_R(\omega_e)}{d\omega_e} = 0 \quad \omega_{e1} \leq \omega_e \leq \omega_{e2}$$

then the value computed by Eq. II-19 will be between  $\omega_{e1}$  and  $\omega_{e2}$ ; not necessarily midway between. In this case, very small changes in the amplitudes of  $S_s(\omega_e)$  and  $R(\omega_e)$ , while not affecting the value of

$$E_s = \int S_s(\omega) d\omega$$

appreciably, are sufficient to effect the value of  $\omega_{e_{sig}}$  to the extent that it may have any value between  $\omega_{e1}$  and  $\omega_{e2}$ . Since the total value of  $E_s$  has not been significantly changed, all values of  $\omega_{e_{sig}}$  in the range

$\omega_{e_1}$  to  $\omega_{e_1}$  are nearly equally probable. Fortunately, the average ship acts like a narrow band filter and responds only to a narrow range of wave lengths approximately the same length as the ship (i.e.  $LWL \pm 0.25 LWL$ ). Since the period of most ship motions of interest in this study (say 10 - 15 seconds) is long in comparison to most motions of interest in the aircraft (1 - 5 seconds) the value of  $\omega_e$  is not critical. This observation is further supported since Eq. II-19 assures us that the value computed lies in the range of nearly equal probability.

The values used in this study for a 30' wave condition, 630' ship moving at 25 knots into head seas were:

$$\text{heave amplitude} = 18.035$$

$$\text{velocity amplitude} = 12.320$$

$$\omega_{e_{SIG}} = 0.679$$

$$\text{or since wave length } \lambda = 203/\omega^2$$

$$\lambda_{SIG} = 449' = 0.70 \times LWL$$

Additional results are shown in Table II-4.

TABLE II-4

		<u>SHIP CASE</u>	
		I	II
$V_s$ , SHIP VELOCITY, KNOTS		25	25
$\eta_3$ , MAX. HEAVE AMPLITUDE <sup>(1)</sup> , FT		18.035	11.207
$\eta_5$ , MAX. PITCH AMPLITUDE <sup>(1)</sup> , DEG		5.492	3.162
$\mu_e$ , NON DIMENSIONAL ENCOUNTER FREQ. <sup>(2)</sup>		3.45	3.45
$\delta'$ , PHASE ANGLE OF PITCH WITH RESPECT TO HEAVE, RAD		.698	1.357
$X_{i_s}$ } COORDINATES OF SHIP	FT	0	0
$Z_{i_s}$ } CG AT $t = 0$			
$X_{i_{LR}}$ } COORDINATES OF LANDING	FT	(3)	(3)
$Z_{i_{LR}}$ } AREA, $t = 0$			
$h_{1/3}$ SIGNIFICANT WAVE HEIGHT, FT		30	20
SS SEA STATE		7	6

(1) Significant (i.e., average of one-third highest) values. In the program, the parameter is the ratio of the "amplitude"  $\eta_3$  to 1/2 the "height"  $h_{1/3}$ , thus  $18.053/15=1.21$ .

(2)  $\mu_e = \omega_e \sqrt{L/g}$  where  $L$  is the ship length.

(3) Three locations were studied: midships and  $\pm 100'$  from midships, all 26.5' above W.L.

### III. AIRCRAFT MOTIONS

As is well known, the equations of motion for an aircraft can be written in a variety of forms, each form more suitable to the solution of a specific problem (such as stability, response to gusts, control response, or automatic control design), and each form subject to advantages and limitations. In the past, most stability and control work has been accomplished with the linearized equations of small disturbance theory. (Reference 4, section 4-14).

Since the stability of the aircraft being studied has already been demonstrated, stability for normal flight conditions is not a primary consideration. Thus, the force and moment equations themselves are used in this study. The principal advantages in using the force and moment equations lies in the ability to handle (a) large angular displacements (e.g. greater than  $10^\circ$ ), (b) non-linear ground and air reactions, and (c) the coupling reactions between longitudinal and lateral forces.

Finally, the computer has removed most of the practical difficulties in the solution of the governing non-linear coupled equations.

The generalized equations of motion of an aircraft, which are used in this simulation, are given in Appendix B. These equations are identical to the equations presented in Reference 4, Section 4, except for the addition of the force and moment equation for the landing gear. They are

generalized in the sense that they apply to a configuration consisting of any combination of the following components:

- (a) Single rotor
- (b) Two rotors in tandem rotor configuration
- (c) Fuselage
- (d) Horizontal tail
- (e) Vertical tail
- (f) Tail rotor
- (g) Propellers or jet engine
- (h) Lift engine or deflectable thrust
- (i) Wings
- (j) Various stabilization devices

For the present simulation, which is limited to the pitch plane, the governing equations of motion are:

The X-Force Equation:

$$X = (X)_F + (X)_{FUS} + (X)_W + (X)_T + (X)_{VT} + (X)_{TR_{i=1}} + \dots$$

$$\sum_{i=1}^n (X)_F - W \sin \theta - \frac{W}{g}(\dot{\theta} + \theta \omega) = 0 \quad \text{III-1}$$

where

$$(X)_F = L_F \sin(\alpha - \epsilon_F) - D_F \cos(\alpha - \epsilon_F)$$

$$(X)_{FUS} = L_{FUS} \sin(\alpha - \epsilon_{FUS}) - D_{FUS} \cos(\alpha - \epsilon_{FUS}) + F_{LG}$$

$$(X)_W = L_W \sin(\alpha - \epsilon_W) - D_W \cos(\alpha - \epsilon_W)$$

$$(X)_T = L_T \sin(\alpha - \epsilon_T) - D_T \cos(\alpha - \epsilon_T)$$

$$(X)_{VT} = -D_{VT} \cos(\alpha - \epsilon_{VT})$$



$$(X)_{TR} = -D_{TR} \cos(\alpha - \epsilon_{TR})$$

$$(X)_{P_i} = T_{P_i} \cos i_{P_i} - N_{P_i} \sin i_{P_i}$$

The Z-Force Equation:

$$Z = (Z)_F + (Z)_{FUS} + (Z)_W + (Z)_T + (Z)_{VT} + (Z)_{TR_{i=1}} +$$

$$\sum_{i=1}^n (Z)_{P_i} + W \cos \theta - \frac{W}{g} (\ddot{w} - \dot{\theta} u) = 0$$

III-2

where

$$(Z)_F = -D_F \sin(\alpha - \epsilon_F) + L_F \cos(\alpha - \epsilon_F)$$

$$(Z)_{FUS} = -D_{FUS} \sin(\alpha - \epsilon_{FUS}) + L_{FUS} \cos(\alpha - \epsilon_{FUS})$$

$$(Z)_W = D_W \sin(\alpha - \epsilon_W) + L_W \cos(\alpha - \epsilon_W)$$

$$(Z)_T = -D_T \sin(\alpha - \epsilon_T) + L_T \cos(\alpha - \epsilon_T)$$

$$(Z)_{VT} = -D_{VT} \sin(\alpha - \epsilon_{VT})$$

$$(Z)_{TR} = -D_{TR} \sin(\alpha - \epsilon_{TR})$$

$$(Z)_{P_i} = -T_{P_i} \sin i_{P_i} + N_{P_i} \cos i_{P_i}$$

The Pitching Moment Equation:

$$M = \sum_{i=1}^n (M)_i = \sum_{i=1}^n (X)_i \hat{z}_i + (M_O)_i + M_I$$

and

$$M = (X)_F \hat{z}_F - (Z)_F \hat{x}_F + (X)_W \hat{z}_W - (Z)_W \hat{x}_W + (X)_T \hat{z}_T -$$

III-3

$$(Z)_T \hat{x}_T + (X)_{VT} \hat{z}_{VT} - (Z)_{VT} \hat{x}_{VT} + (X)_{TR} \hat{z}_{TR} -$$

$$(Z)_{TR} \hat{x}_{TR} + \sum_{i=1}^n [(X)_{P_i} \hat{z}_{P_i} - (Z)_{P_i} \hat{x}_{P_i}] + M_{P_i} + M_{FUS} +$$

$$M_{HUB_F} + Q_{TR} - \partial I_{yy} = 0$$

III-3

TABLE III-1

SIGN CONVENTION

	<u>POSITIVE</u>
x	FWD
z	DOWN
$\theta$	NOSE UP
$\dot{x}$	FWD
$\dot{z}$	DOWN
$\dot{\theta}$	NOSE UP
$\alpha$	pos. nose up (cw rotation from wind vector to x-axis)
$\beta$	pos. if L.E. swash plate is down
$\gamma$	UP
$\epsilon_{FUS}, \epsilon_T$	pos. if increases local $\alpha$
$l_x$	FWD of CG. (in pos x-direction)
$l_z$	BELOW CG. (in pos z-direction)

Forces and moments as shown in Figure III-1.



where the notation is given in Figure III-1 and Appendix B. The sign conventions are given in Figure III-1 and Table III-1. Assuming that the external forces and moments are known, these equations represent three equations in the three unknowns  $u$ ,  $w$ , and  $\theta$  and can thus, in principle, be solved. In the present simulation this is accomplished numerically using a fourth-order Runge Kutta integration scheme.

The method of determining the forces and moments of the governing equations for the particular aircraft considered in this simulation, the CH-53D, is given below. Further information applicable to the CH-53D aircraft is given in Appendix C. These methods are considered in the following order:

Landing Gear Forces

Rotor Forces, Angle of Attack, and Engine Power

Rotor Hub Moment

Fuselage Forces and Moments

Tail Forces and Moments

Interference Angles

Automatic Flight Control System (AFCS)

The Ground Effect

#### III-1 Landing Gear Forces

(1) Pneumatic shock strut Isothermal compression is assumed. Maximum stroke on both gears is 12". In the following formula, stroke is measured from the fully

compressed position. The stroke vs. pressure characteristics (Ref. 7-9) for the landing gears are as follows:

		PRESSURE, psi	
	<u>STROKE</u>	<u>MAIN</u>	<u>NOSE</u>
STATIC	2"	850	
67° EXT	8"		365
EXTENDED	12"	187	275

Nose Gear Determine cylinder length:

$$P_{67^\circ} L = P_e (L + 4)$$

$$365 L = 275 (L + 4)$$

$$L = 12.22" \text{ from } 67^\circ \text{ EXT.}$$

Pressure, fully compressed

$$P_c L_c = P_e L_e$$

$$P_c = \frac{P_e L_e}{L_c} = \frac{275 \times 16.22}{4.22} = 1060 \text{ psi}$$

Load at any stroke

$$\text{LOAD} = PA = \frac{P_c L_c A}{S + L_c}$$

$$= \frac{1060 \times 4.22 \times 19.36}{S + L_c}$$

$$= \frac{86600}{S + 4.22} \quad \text{PER STRUT}$$

$$= \frac{173200}{S + 7.22} \quad \text{TOTAL (2 STRUTS)}$$

III-4

MAIN GEAR Determine cylinder length:

$$P_s L = P_e (L + 10)$$

$$850 L = 187 (L + 10)$$

$$L = 2.825 \text{ from static}$$

$$= .825 \text{ from compressed.}$$

Pressure, fully compressed

$$P_e = \frac{P_s L_s}{L_c} = \frac{850 \times 2.825}{.825} = 2915 \text{ psi}$$

Load at any stroke

$$\text{LOAD} = PA = \frac{P_c L_c A}{S + L_c} = \frac{2915 \times .825 \times 19.36}{S + 0.825}$$

$$= \frac{46500}{S + 0.825} \quad \text{PER STRUT}$$

$$= \frac{93000}{S + 0.825} \quad \text{TOTAL (2 STRUTS)}$$

III-5

In the simulation model all forces in the lateral plane are assumed to be zero, hence the load equations for both nose and main gear are multiplied by 2 to give total load.

(2) Damping Damping is provided by the restriction of the flow of oil through an orifice with a sharp edged entrance on the lower side, and a rounded entrance (radius = .78 x plate thickness) on the upper side. Damping is in both directions, adding to the pneumatic force as the strut is compressed and subtracting as it extends.

The properties of the oil used in the shock strut are not known and no information concerning damping characteristics of the shock strut is given in the reports listed among the

references. For the purposes of this study, it was assumed that the oil had the following properties (at 60°F)

$$\mu = 500 \text{ centipoises (absolute viscosity)}$$

$$S = .88 \quad (\text{specific gravity})$$

$$\text{Thus } \rho = .85 \times 62.4 = 55 \text{ \#/ft}^3$$

$$\mu = 500 \times 2.09 \times 10^{-5} \approx 10^{-2} \text{ slugs/ft-sec}$$

$$R = \text{Reynolds number}$$

$$= \frac{V d_o \rho}{\mu} \text{ where } d_o = \text{orifice diameter}$$

$$V = \text{velocity through orifice}$$

$$d_o = 0.75"$$

$$d_i = 4.122"$$

In terms of piston speed  $V'$

$$R = \left( \frac{4.122}{.75} \right) \left( \frac{4.122}{12} \right) \left( \frac{55}{10^{-2}} \right) V'$$

$$\approx 10^4 V'$$

$V'$  is the compression or extension rate of the shock strut and is equal to the sink or rebound rate (in ft/sec) of the aircraft. Values of  $V'$  for which the damping force is significant (say more than 1/10th the air compression force) are above one foot per second, hence the Reynolds number in all cases of interest is above  $10^4$ . In the range  $10^4 \leq R \leq 10^6$ , the discharge coefficient  $C$  for a diameter ratio  $d_o/d_i = .75/4.122 = 0.18$  is approximately constant and equal to 0.6 (see Ref. 14). Solving the standard flow equation

$$Q = C A_n \sqrt{\frac{2g\Delta P}{\rho}}$$

III-6

$$\text{for damping force (D.F.)} = \Delta P \times A_c \text{ (psi} \times \text{in}^2)$$

in terms of  $q = V_c A_c$  (in/sec x in<sup>2</sup>)

where  $V_c$  is the piston or stroke velocity, in/sec, for  $C = 0.60$

and  $\rho = 55 \text{ #/ft}^3 = .032 \text{ #/in}^3$

$$\begin{aligned} D.F. &= \frac{\rho A_c}{2g C^2} \left( \frac{A_c}{A_n} \right)^2 V_c^2 \\ &= \frac{.032 \times 19.36}{2 \times 386.4 \times 0.60^2} \left( \frac{d_1}{d_o} \right)^2 V_c^2 \\ &= \frac{.62}{278} \left( \frac{4.122}{.75} \right)^2 V_c^2 \\ &= \frac{.062 \times 30.2}{278} V_c^2 = .067 V_c^2 \quad (V_c \text{ in in/sec}) \\ &= 9.65 V_c^2 \quad (V_c \text{ in ft/sec}) \\ &\approx 10 V_c^2 \end{aligned}$$

There is no mention of a metering pin in the available literature, except obliquely in Ref. 8. However, this extremely low value of the damping factor indicates that one must be present. In terms of the ratio  $r_{mn}$  of the metering pin diameter,  $d_m$ , to the orifice diameter,  $d_o$ , the damping factor can be written as:

$$\begin{aligned} D.F. &= 10 [1 - r_{mn}^2]^{-1} V_c^2 \\ &= C_D V_c^2 \end{aligned}$$

where  $C_D$  is the damping constant and  $r_{mn} = \frac{d_m}{d_o}$ .

For practical values of  $r_{mn}$ ,  $C_D$  has the following values:

$r_{mn}$	$C_D$
.9	52.7
.95	100
.975	200
.985	360



For a metering pin diameter  $d_m = 0.950 d_n$ , the clearance in the orifice hole between the sides of the pin and the orifice edge is  $(1/2)(.050) (.75) = (.0250) (.75) = .01875"$ , which is slightly larger than  $1/64"$ . In the simulation, the damping constant ( $C_D$ ) was chosen to correspond to 100. Thus, for a sink (or rebound) speed of 10 fps, the damping force is  $D.F. = 100 \times 100 = 10000\#$  (or  $-10,000\#$ ).

Both the viscosity and density of the oil are functions of the temperatures. The variations in viscosity should have no effect. Although the variation is quite large, viscosity effects only the Reynolds number, which in turn effects the orifice coefficient  $C$ . But in the range of interest,  $C$  is a constant. Density is less affected by temperature, there being a drop of perhaps 10% as the temperature changes from 0 to  $100^\circ F$ , and in view of the other uncertainties of the damping force calculation, this can be ignored.

Usually, metering pins taper from the base to a point near the end where the pin flares into some sort of a bulb, so that maximum orifice constriction occurs at either end of the stroke. This variation was ignored.

Thus, for the present simulation, the damping force is assumed to be given by the expression:

$$D.F. = \pm 100 v_c^2 \quad \text{III-7}$$

The plus sign applies when the strut is extending, since the damping force is downward.

(3) Tire Forces In the interest of simplicity, tire forces are not included in this version of the simulation. The inclusion of the tire would add a degree of freedom to each landing gear. This assumption can be justified by noting that the work done by the tire is small in comparison to the work done by the shock strut. Further, the work done by the tire has a strong effect on the dynamics of the aircraft. Since the unsprung mass begins moving at a somewhat later time, the real effect on landing gear forces, as far as the aircraft is concerned, is to delay the build-up of forces during the landing. However, the overall effect is small in cases where the unsprung mass is small and hence can be ignored.

### III-2 Rotor Forces, Angle of Attack and Engine Power

Assuming that the thrust is known, the following parameters can be determined:

- (a) Thrust coefficient  $C_{T_R}$
- (b) Collective pitch  $\theta_{.75}$  and  $\lambda$  for minimum profile drag
- (c) Torque coefficient  $C_{Q_R}$  and Torque  $Q_R$  required
- (d) Power required  $P_R$
- (e) Power ratio (PR) - required-to-available
- (f) Rotor angle of attack  $\alpha_c$
- (g)  $H$  - Force
- (h) Rotor lift and drag.

The computations are accomplished in the following order:

a) Thrust coefficient

$$C_T = \frac{W (T_R/W)}{\pi R^2 (\Omega R)^2} \quad \text{III-8}$$

b) Collective pitch setting for minimum torque

If Eqs. 72 and 74 of Reference 13 are added, the resulting expression is the net torque at the rotor shaft and has the form:

$$C_Q = K_1 \theta_o^2 + K_2 \theta_o + K_3 \lambda \theta_o + K_4 \lambda^2 + K_5 \lambda + K_6 \quad \text{III-9}$$

where the coefficients  $K_i$  are lengthy expressions which are defined in sub-section 3. Eq. 69 of Ref. 12 gives the thrust coefficient  $C_T$  in terms of  $\lambda$  and  $\theta_o$ :

$$\frac{C_T}{\sigma_a} = t_{3,1} \lambda + t_{3,2} \theta_o + t_{3,3} (\theta_o + \theta_t) \quad \text{III-10}$$

$\theta_o$  is the blade pitch at the root (collective pitch) and  $\theta_t$  is the blade twist. Since  $C_T$  is known, Eq. III-10 may be solved for  $\lambda$  which may be substituted in III-9. This yields a quadratic in  $\theta_o$ . Taking the first derivative and setting it equal to zero gives:

$$\theta_o = \frac{D}{N} \quad \text{III-11}$$

$$\text{where } D = [a(t_{4,2} + t_{4,3}) - \delta_2(t_{5,6} + t_{5,7})] - \delta_1(t_{5,3} + t_{5,4}) \\ + (at_{4,5} - \delta_2 t_{5,9}) \theta_t + 2(at_{4,6} - \delta_2 t_{5,10}) \quad \text{III-12}$$

$$\text{and } N = 2\delta_2(t_{5,8} + t_{5,9} + t_{5,10}) - 2a(t_{4,4} + t_{4,5} + t_{4,6}) \\ + \frac{t_{3,2} + t_{3,3}}{t_{3,1}} [\delta_2(t_{5,6} + t_{5,7}) - a(t_{4,2} + t_{4,3})] \quad \text{III-13}$$

In these expressions

$t_{ij}$  are the Bailey coefficients, Ref. 13

$\delta_i$  are the coefficients in the section drag equation

$$c_{d_0} = \delta_0 + \delta_1 \alpha_r + \delta_2 \alpha_r^2 \quad (\text{Ref. 4 - 12})$$

$\alpha$  is the slope of the section lift curve

and  $\theta_t$  is the blade twist.

Since the second derivative of  $C_Q$  is positive for a given value of  $C_T$ , Eq. III-11 gives the value of  $\theta_0$  for minimum torque. With this value of  $\theta_0$ , Eq. III-10 may be solved for  $\lambda$ .

### c) Torque coefficient $C_Q$

The torque coefficient may be calculated by means of Eq. III-9 using the values of  $\theta_0$  obtained from Eq. III-11, and  $\lambda$  from III-10.

The coefficients of III-9 are

$$\left. \begin{aligned} K_1 &= (\delta_2 t_{5,8} - \alpha t_{4,4}) + (\delta_2 t_{5,9} - \alpha t_{4,5}) + \delta_2 t_{5,10} \\ K_2 &= \delta_1 t_{5,3} + \delta_1 t_{5,4} + (\delta_2 t_{5,9} - \alpha t_{4,5}) \theta_t + 2 \alpha t_{4,6} \\ K_3 &= (\delta_2 t_{5,6} - \alpha t_{4,2}) + (\delta_2 t_{5,7} - \alpha t_{4,3}) \\ K_4 &= (\delta_2 t_{5,5} - \alpha t_{4,1}) \\ K_5 &= \delta_1 t_{5,2} + (\delta_2 t_{5,7} - \alpha t_{4,3}) \theta_t \\ K_6 &= \delta_0 t_{5,1} + \delta_1 t_{5,4} \theta_t + \delta_2 t_{5,10} \theta_t^2 \end{aligned} \right\} \quad \text{III-9a}$$

Torque is given by the expression

$$Q_R = C_Q (\pi R^2 \rho (\Omega R)^2 R) \quad \text{III-14}$$

$$\text{d) } \underline{\text{Power required}} \quad P_R = \frac{Q_R \Omega}{550} \quad \text{shp.} \quad \text{III-15}$$

Maximum power for the CH-53D, a function of temperature ( $^{\circ}\text{C}$ ), is taken from Figures 4.5 and 4.6 in Ref. 6.

$$\begin{aligned} P_{\max} &= 6422 \quad \text{shp} \quad t \leq 15^{\circ}\text{C} \\ &= 6858 - 29.1t \quad 15^{\circ}\text{C} > t > 59^{\circ}\text{C} \quad @ 100\% \text{ NF} \\ &= 5140 \quad t \geq 59^{\circ}\text{C} \end{aligned} \quad \text{III-16}$$

e) The rotor angle of attack

Eq. 70 of Ref. 12 yields:

$$\alpha_c = \arctan \left[ \frac{\lambda}{\mu} + \frac{C_T}{2\mu(\mu^2 + \lambda^2)^{1/2}} \right] \quad \text{III-17}$$

f) The H Force The profile drag-lift ratio is expressed as

$$\left( \frac{D}{L} \right)_c = \frac{\sigma a}{2\mu} \left[ \frac{\delta_0}{a} t_{6,1} + \frac{\delta_1}{a} (t_{6,2} \lambda + t_{6,3} \theta^{.75}) + \frac{\delta_2}{a} (t_{6,5} \lambda^2 + t_{6,6} \lambda \theta^{.75} + t_{6,8} \theta^{2.75}) \right]$$

It is also equal to

$$\left( \frac{D}{L} \right)_c = \frac{H \cos \alpha_c - T \sin \alpha_c}{T \cos \alpha_c - H \sin \alpha_c}$$

Thus

$$H = \frac{-T(\sin \alpha_c + \frac{D}{L} \cos \alpha_c)}{\cos \alpha_c + \frac{D}{L} \sin \alpha_c} \quad \text{III-18}$$

g) Rotor lift and drag The rotor lift and drag:

$$L_F = T \cos \alpha_c - H \sin \alpha_c \quad \text{III-19}$$

$$D_F = H \cos \alpha_c - T \sin \alpha_c \quad \text{III-20}$$

III-3 The Rotor Hub Moment

The moment due to flapping hinge offset is given in Ref. 5 (p. 49) as

$$M_{fh} = K(a_1 - B_{1S}) - \left( \frac{a_0}{17R} \right) \left( \frac{\dot{\theta}}{\omega} \right) \left[ 1 - \left( \frac{e}{R} \right)^3 \right] \quad \text{III-21}$$

The term  $a_1$  is the backward tilt in flapping and is given by the expression (Eq. 65, Ref. 12)

$$a_1 = t_{1,4} \lambda + t_{1,5} \theta_0 + t_{1,6} \theta_1 = t_{1,4} \lambda + t_{1,5} \theta_{.75}$$

In most forward flight conditions  $a_1 < B_{1S}$ , where  $B_{1S}$  is the forward cyclic, so the first term is negative, pitching nose down. The term  $K$  is the hub pitch constant, ft-lbs/deg:

$$K = \frac{eb\Omega^2 M_s}{2}$$

The second term adds to the first if  $\dot{\theta}_f$  is negative (pitching nose down), but the whole term is usually small and can be ignored. Thus the hub moment is

$$M_H = \frac{eb\Omega^2 M_s}{2} (a_1 - B_{1S}) \quad \text{III-22}$$

For the CH-53D:

$e = 2.0$
$b = 6$
$M_s = 184$
$\Omega = 22$

Thus

$$M_H = \frac{2 \times 6 \times 22^2 \times 184}{2} (a_1 - B_{1S}) \quad \text{III-23}$$

$$= 5.35 \times 10^5 (a_1 - B_{1S})$$

The value of  $a_1 - B_{1S}$  may be as high as  $-3^\circ$  so  $M_H$  might be about  $-29000$  ft-lbs. for the CH-53D.

#### III-4 Fuselage Forces and Moments

The equations for fuselage lift, drag and pitching moment are based on wind tunnel data for the CH-53D presented in Refs. 10 and 11, and shown graphically in Ref. 5. A

polynomial was fitted to the data for the range of fuselage angle of attack from  $-16^\circ$  to  $+16^\circ$ . The resulting equations are

$$\text{LIFT} \quad L_F/q = 15.0 + 487 \alpha_F \quad \text{III-24}$$

$$\text{DRAG} \quad D_F/q = 41.696 - 11.45 \alpha_F + 8.423 \alpha_F^2 \quad \text{III-25}$$

$$\text{FOR THE RANGE} \quad -20^\circ \leq \alpha_F \leq +20^\circ$$

$$\text{MOMENT} \quad M_F/q = -450 \quad -20 \leq \alpha_F \leq -12 \quad \text{III-26}$$

$$M_F/q = 58.8 \alpha_F \quad -12 \leq \alpha_F \leq +16 \quad \text{III-27}$$

$$M_F/q = 1000 \quad +16 \leq \alpha_F \leq +20 \quad \text{III-28}$$

The lift data is for fuselage and tail. The tail incidence equals  $3^\circ$ . The drag data is for the complete aircraft, including rotor head, antenna, cooling losses, etc. The moment data is for the complete aircraft, minus the tail. The full-scale data shown in Ref. 5 has been transferred to a c.g. at station 386 and a waterline station of 161.4.

### III-5 Tail Forces and Moments

The horizontal tail lift coefficient is

$$C_{L_T} = a (\alpha_T - \alpha_0) \quad \text{III-29.1}$$

where the angle of attack  $\alpha_T$  is

$$\alpha_T = \theta_f + i_T - \epsilon_T + \epsilon_p \quad \text{III-29.2}$$

and  $\alpha_0$  is the zero lift angle. The slope of the lift curve

$$a = C_{L_T} / \alpha$$

$$a = \frac{\pi / R}{1 + \sqrt{\left(\frac{R-1}{s_0}\right)^2 + 1}} \quad \text{III-29.3}$$

III-17

where  $\alpha_0$  is the lift curve slope for the airfoil section and is taken to equal 5.73. For an aspect ratio equal to 2.5,  $a = 2.88/\text{RAD}$  or  $0.05/\text{DEG}$ .

$\alpha_T$ , the angle of attack of the tail, is equal to the fuselage pitch angle  $\theta_f$ , the tail plane incidence angle  $i_T$ , the downwash interference angle  $\epsilon_T$  and an additional angle of attack change produced by the fuselage pitching motion and equal to

$$\epsilon_p = \frac{\dot{\theta}_f l_{x_T}}{V}$$

where  $l_{x_T}$  is the distance of the tail to the c.g. The angle  $\epsilon_T$  is discussed in the next section.

$$C_{M_T} = l_{x_T} C_{L_T}$$

III-29.4

Tail drag is a part of fuselage drag and included in Equation III-25.

### III-6 Interference Angles

The generalized equations of motion contain the downwash interference angles

(1)  $\epsilon_{FUS}$  Rotor on fuselage III-30

(2)  $\epsilon_T$  Rotor on tail III-31

which apply in this case, as well as several others ( $\epsilon_p$ ,  $\epsilon_R$ ,  $\epsilon_W$ ,  $\epsilon_{VT}$ , and  $\epsilon_{TR}$ ) which do not apply. The formulas  $\epsilon_{FUS}$  and  $\epsilon_T$  are taken from Ref. 4, section 5.

$$\epsilon_{FUS} = \tan \alpha_c - \frac{1}{b}$$

$$\epsilon_T = \tan \alpha_c - \frac{1}{b}$$



Some consideration was given to the use of the formula

$$\epsilon_T = K_{FT} \left[ \tan \alpha_c - \frac{\lambda}{\mu} \right]$$

where  $K_{FT}$  is a function of the rotor wake angle

$$x = a_1 + \tan^{-1} \left[ -\frac{\mu}{\lambda} \right]$$

The functional relationship is shown in Ref. 4, Section 5, Figure 1. For a range of  $x$  from  $40^\circ$  to  $80^\circ$ ,  $K_{FR}$  (and  $K_{FT}$ ) ranges from 0.37 to 1.46. Since no information was available on the wake angle  $x$  for the flight conditions of interest, it was decided to defer this question.

### III-7 Automatic Flight Control System (AFCS)

The simulation model of the AFCS is mathematically identical, except in one minor respect, to the AFCS installed in the CH-53D aircraft. The exception is due to the finite difference nature of the AFCS simulation which delays the feedback one time step. The pitch transfer function of the AFCS feedback loop is derived as follows:

$$B_{1s} = K_R \dot{\theta}_f + K_P \theta_f$$

where  $B_{1s}$  is the cyclic pitch of the swash plate

$\theta_f$  is the pitch angle of the fuselage and  $K_R$  and  $K_P$  are the rate and proportional gains. The transfer function is

$$\frac{B_{1s}}{\theta_f} = K_P (TS + 1)$$

where  $T = \left( \frac{K_R}{K_P} \right)$ . The feedback gains for optimum performance have been derived in Reference 5, for the aircraft system

$$\frac{B_{1s}}{\theta_f} = .4(S + 2) \text{ "cyclic/"fuselage}$$

The question of time delay referred to above is of interest. If the AFCS simulation were described by continuous (analytic) differential equations, the time delay could be represented by the transfer function  $e^{-st}$  and its effect could be readily determined by the various techniques of control system analysis, say by a Nyquist plot. However, when the AFCS simulation employs difference equations, the control feedback is delayed, i.e.,

$$(B_{1s})_{t=t_n + T} = K_R (\dot{\theta}_f)_{t=t_n} + K_P (\theta_f)_{t=t_n}$$

and the value of  $B_{1s}$  is calculated from the values of  $\dot{\theta}_f$  and  $\theta_f$  at the previous time step.

Figure III-2 shows the effect of step size. The curves trace the motion in the x-z plane. At the end of seven intervals (8.4 seconds) the coordinates are:

<u>h</u>	<u>8.4/h</u>	<u>x</u>	<u>z</u>
.010	840	.20	3.85
.025	336	.70	3.00
.050	168	1.02	3.38
.100*	84	2.30	4.10
.150*	60	4.90	6.90

\*curves not shown in Figure III-2.

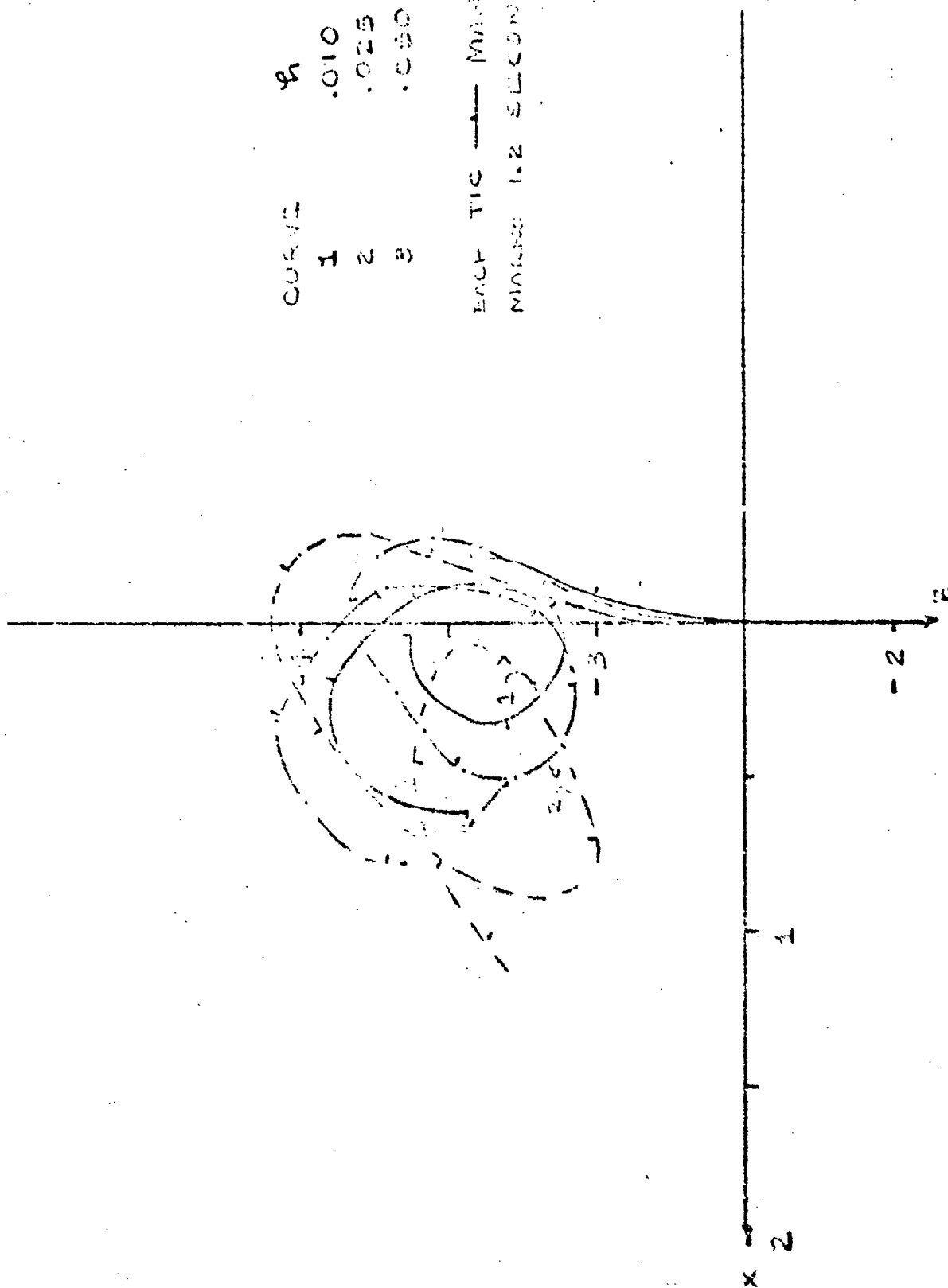


Figure III-2 Effect of Integration Step Size on Hover Behavior - CH-53 Helicopter

It can be seen from the results above and from Figure III-2 that the error due to the finite difference representation of the AFCS is not large if the step size is less than .050, for the hover conditions only. Conservatively assuming that all the differences in Figure III-2 were due to the delayed feedback error, then the error incurred in increasing the step size from .010 to .025 would be .

$$\epsilon_A = \frac{\sqrt{(.70-.20)^2 + (3.85-3.00)^2}}{336} = \frac{.987}{336} \\ = .00294$$

which is less than  $h$  but greater than  $h^2$ . It can be shown by the examination of a simplified version of the equations of motion that the delayed feedback error is of order  $Nh$ , but not  $Nh^2$ , where  $N$  is a positive number which is a function of the  $x$  force.

### III-8 The Ground Effect

Figure III-3 shows the variation of the ground effect parameter

$$A = \frac{\text{rotor thrust in ground effect}}{\text{rotor thrust out of ground effect}}$$

as a function of the ratio  $z/R$

$z$  = distance of rotor head above ground

$R$  = rotor radius

in terms of the ratio  $V/U_0$

$V$  = forward speed ( $-U$ )

$$U_0 = \left( \frac{T}{2 \pi \rho R^2 B^2} \right)^{1/2} = \text{induced velocity in hover}$$

$V$  = tip loss factor = 0.97

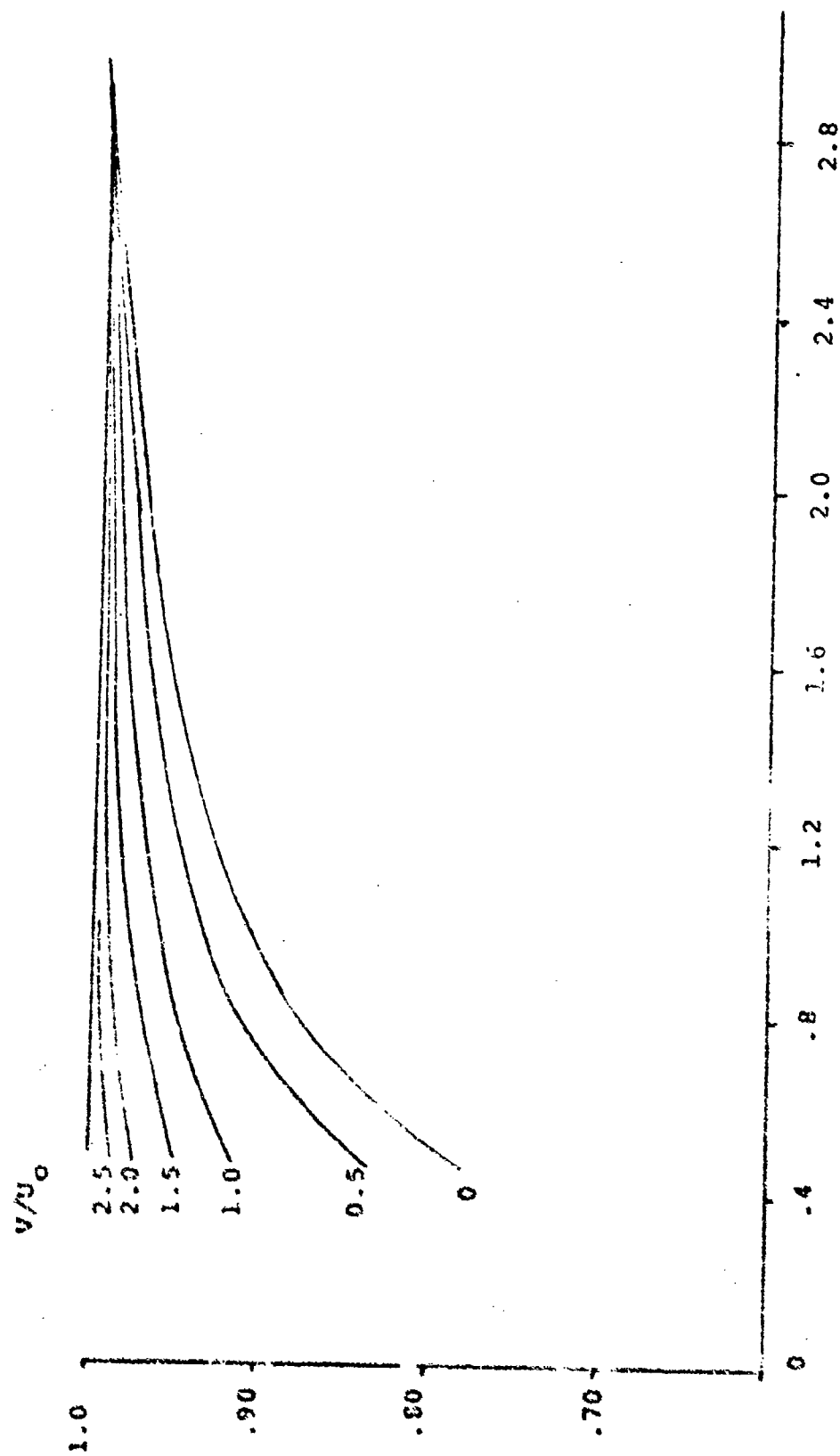


Figure III-3 Ground Effect Parameter A vs.  $z/R$

For  $T = W = 33500$

$$U_o = \left( \frac{33500}{(2\pi)(0.00238)(36)^2(0.97)^2} \right)^{1/2}$$
$$= 76.7 \text{ fps}$$

Since the launch/recovery operations are limited to an initial or terminal speed of about 25 knots ship velocity and 45 knots wind velocity (the maximum allowed for CH-53D wind-up)

$$V = 70 \text{ knots at take-off}$$

$$= 118 \text{ fps.}$$

$$\text{and } V/U_o = 1.55.$$

The parameter  $\Lambda$  gives the reduction in thrust due to ground effect. Ground effect will alter each of the variables computed above, since the effective thrust is reduced. Since the LARC-I calculation begins with a value of thrust (in terms of  $T_R/W$ ), and this value is multiplied by  $\Lambda$  (as a function of  $F/U_o$ ), then the values for  $C_T$ ,  $C_{Q_R}$ ,  $\theta_o$ ,  $\alpha_c$ ,  $H$ ,  $L_F$  and  $D_F$  all include the ground effect. However, it should be noted that the unbalanced moment due to a rotor being only partially in the ground effect, as when the rotor extends over the edge of the deck, is not included.

#### IV. GENERAL DESCRIPTION OF THE LARC-I SIMULATION

The LARC-I (LAunch and Recovery Capability) simulation provides the necessary coupling between the motions of the ship, discussed in Section II, and the aircraft motions, discussed in Section III. The motions of the ship are confined to the x-z plane (i.e., to heave, pitch and longitudinal velocity) and those of the aircraft to longitudinal and vertical velocity and to pitching about the aircraft center of gravity.

Physical interaction of the ship and the aircraft occurs only through the landing gear forces and moment:

$$Z_{LG} = - (F_{LG_N} + F_{LG_M})$$

$$X_{LG} = \text{brake force} = \pm \mu_F Z_{LG}$$

$$M_{LG} = F_{LG_N} X_{LG_N} + F_{LG_M} X_{LG_M}$$

( $\mu_F$  = coefficient of friction, usually 0.3)

$F_{LG_N}$  and  $F_{LG_M}$  are, respectively, functions of  $S_N$  and  $S_M$ , the extension or stroke of the nose and main landing gear shock strut cylinders which are in turn functions of the relative aircraft and ship positions. The equations for stroke are shown in Appendix C, page C-1. It is only in these equations, in the terms  $Z_{i_{LP}}$  (vertical position of the ship two aircraft launch/landing points) and  $\dot{Z}_{i_{LP}}$  (velocity of the same points) that the ship motion interacts with the aircraft and the interaction through the landing gear forces

persists only as long as the aircraft is in contact with the deck.

The variables  $z_{iLP}$  and  $\dot{z}_{iLP}$  are in turn harmonic functions of time which include the coupled heaving and pitching of the ship in response to waves of a random sea. The sea state is, in this simulation, characterized by a single parameter, the significant wave height  $h_{1/3}$  which may take values (crest to trough) of up to 100 feet in sea state 9 conditions.

Further coupling between the ship and the aircraft occurs indirectly through their relative motions. Since both the ship and aircraft motions are referred to a body axis system it is necessary to refer the motion of each vehicle to known fixed inertial axes (see Figure IV-1) in order to obtain the relative motion between the vehicles. The necessary transformation equations are given in Appendix B.

#### IV-1 Structure of the LARC-I Simulation Program

Figure IV-2 is a simplified block diagram of the LARC-I simulation program. The simulation may be described as a fixed control non-steady state model. Each computer run simulates a segment of length T (seconds) during which the program computes the motion at  $N = \frac{T}{\Delta T}$  instants ( $\Delta T$  is the time step) during which all input variables including the controls remain fixed. If all aircraft initial input coordinate constants ( $x_0, \dot{x}_0, z_0, \dot{z}_0, \theta_0, \dot{\theta}_0, t_0$ )





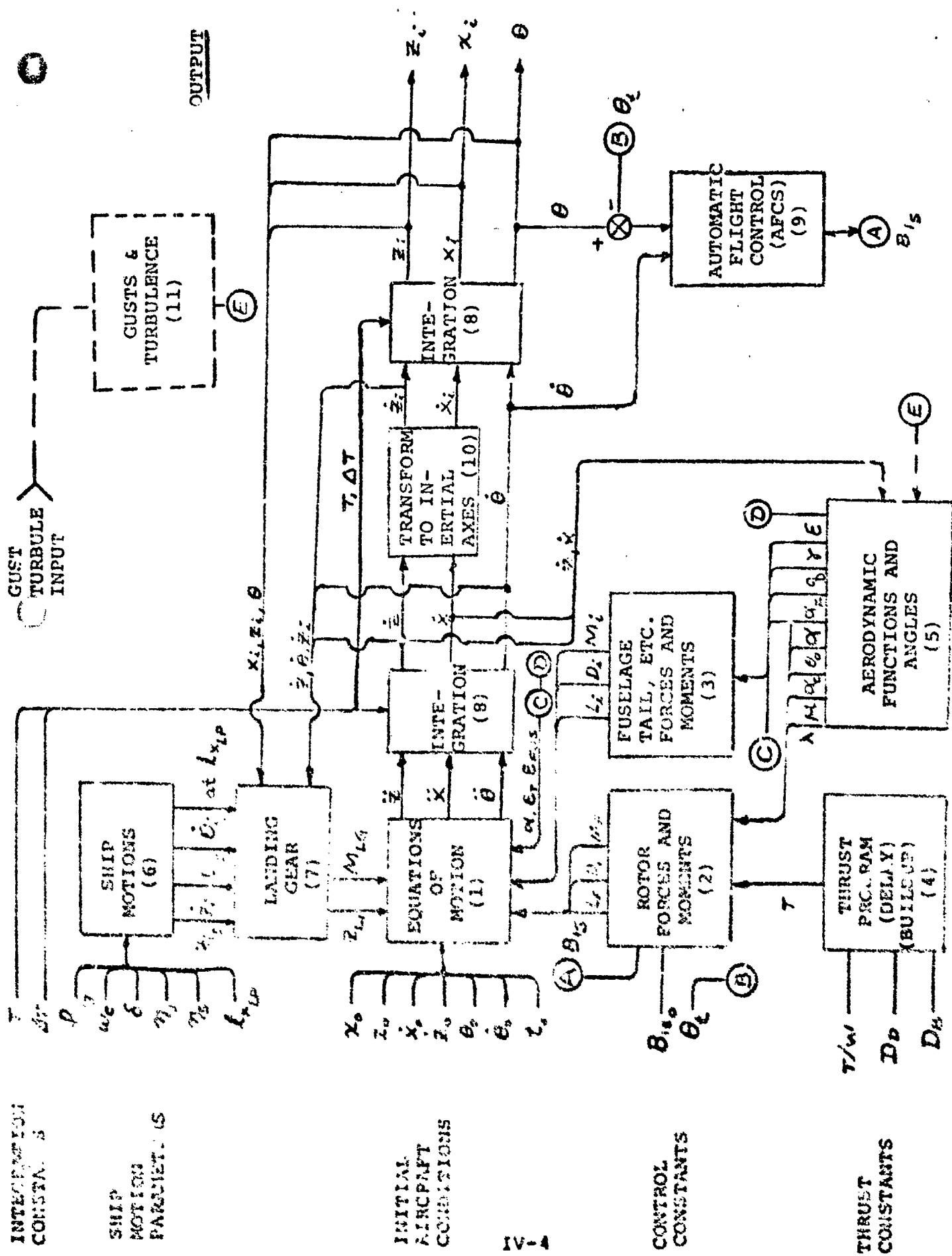


FIGURE IV-2 SIMPLIFIED AIRCRAFT SIMULATION BLOCK DIAGRAM

are set to zero, the aircraft is placed on the deck at a distance  $\lambda_{x_{LP}}$  from the ship's center of gravity.

The sequence of operations for the simulation which uses a fourth-order Runge Kutta scheme to numerically integrate the equations of motion is shown in Figure IV-2. At each time instant  $T_i$  the force and moment modules (2), (3) and (7) compute the forces and moments based on the attitude and velocity of the aircraft at the end of the previous time step,  $T_i - \Delta t$ , and, if the aircraft is in contact with the deck, the position and velocity of the ship, block (7). At each time  $T_i$ , a value of  $B_{\lambda_S}$  is calculated at block 9 based on the value of  $\dot{\theta}_f$  and  $\dot{\theta}_f$  at the previous time  $T_i - \Delta T$ .

If the thrust/weight parameter ( $T/W$ ) is greater than one, the aircraft will ascend, if less than one, the aircraft will descend. The thrust may be delayed  $D_B$  seconds by assigning a value to the delay parameter  $D_D$ . This allows the take-off sequence to start at some specified point in the ship motion cycle. Experience has shown that the parameter  $D_B$  is less important in the landing sequence. Also, the thrust may be specified to increase linearly from zero to  $(T/W)W$  pounds in  $D_B$  seconds for use primarily in the take-off sequence.

The initial longitudinal cyclic,  $B_{\lambda_S}$  may be specified. However, the Automatic Flight Control System will continuously vary  $B_{\lambda_S}$  according to the equation

$$B_{\ell_s} = K_r \dot{\theta}_f + K_p (\theta_f - \theta_t)$$

where  $K_p$  is the proportional gain,  $K_r$  the rate gain,  $\theta_f$  the fuselage pitch angle, and  $B_{\ell_s}$  is the swash plate tilt angle relative to the rotor shaft. The effect of assigning a value to  $B_{\ell_s}$  may reduce the magnitude of a transient motion, since otherwise  $B_{\ell_{s_0}}$  is assumed zero.

The trim pitch angle is the fuselage pitch corresponding to near equilibrium conditions at a thrust equal to  $(T/W)W$  and for all other values of the initial conditions. For a given gross weight and c.g. position,  $\theta_t$  is a function of forward velocity. If  $\theta_t$  is not known for a given velocity, it may be found by a trial and error procedure. Since the LARC program outputs all forces as functions of time, the trim angle for a given speed may be easily approximated using the data from a few runs. Although trim pitch is specified, the aircraft in the simulation is rarely, if ever, in a state of trim, or steady state flight, although the changes about the trim value will not be large. The thrust program is shown in block 4. The integration scheme is shown in block 8. The dashed block 11 refers to the model for gusts or turbulence, which, although very simple, was not included in the present version of the simulation.

#### IV-2 Operation of the LARC-I Program

The following remarks are intended to illustrate the operation of the simulation. They are not intended to

serve as a user manual. Operation of the program is quite simple. A user with no prior knowledge of the program itself, or prior background in programming, can make full independent use of the program after only very brief instructions.

The program is designed for interactive operation on a time-shared computer using a remote terminal. The user communicates by means of the terminal keyboard. The program response is by terminal print-out or graphic display. Equipment used in this project was a Honeywell H-1648 time-shared computer, accessed by various remote terminals such as Tektronix 4010 and Typagraph 3 and DP-30.

The LARC-I program written almost entirely in FORTRAN IV, is essentially machine-independent. Certain output, plotting and file processing routines which are required by the particular interactive graphics terminal (Tektronix 4010) are features of the H-1648 time-shared operating system. Since these features are common to most time-shared systems, operation of the program on computer systems other than the H-1648 can be accomplished with relatively small changes in the coding. Machine independency was a major goal of this project.

All processing on the time-shared system is done using files which contain programs or data; each file is created independently. The files are accessed by reference to file name, either internally in the program or by means of an interactive command. Data files may be created in their

entirety, may be searched for specific elements, or may be changed by altering specified elements or groups of elements.

Subsequent to the creation of the data files referred to below, and to the compilation of the LARC-1 program, the sequence of interactive commands may appear as follows (the user replies are underlined):

TYPE AIRCRAFT TYPE \*\*\* CH53

The user response loads a file entitled "CH53", containing the input data shown in Appendix C. The user may select any data file which he has previously created, each representing a particular aircraft configuration. Elements of this file may not be altered during run time.

\*\* D

The user response loads the parameter file DEFAULT. The values of this file may be changed during run-time in the manner described below

\*\* I

The user response "I" for INPUT, signals the intention of changing values in the parameter file DEFAULT. If this response is not made, the DEFAULT file is loaded as compiled prior to run-time.

PARAMETER CODES - YES OR NO NO

If the user response is "YES", the parameter codes are printed as shown in Table IV-1. These codes assign a number to be used in the responses listed below.

INPUT CODE AND VALUE 21,0.03

The user has assigned a value 0.05 to parameter 21 (the run-time). The user may assign values to any or all of the parameters listed in Table IV-1 in any desired order.

INPUT CODE AND VALUE 25

This response is required to terminate the input sequence following the command "I" and return the program to the command mode.

\*\* T

The user response is "T" for take-off but may be "L" for landing. This initiates the computational sequence which continues uninterrupted for  $N = T/\Delta T$  steps. Both T (run-time) and  $\Delta T$  (time step interval) are parameters and may be altered for each run.

\*\* P

In the present version, the user may command output in tabular form, "P" or graphical form, "G".

HOW MANY CURVES (1 - 3) 3

- 1. - CURVE # \*\* 1
- 2. - CURVE # \*\* 2
- 3. - CURVE # \*\* 10

Here, the user specifies the curves to be printed or displayed graphically in any desired order, in groups of one to three curves. Appendix A contains examples of the printed output. Table IV-2 lists the output parameters which may be printed on command. These 29 files are loaded in their entirety each time the program executes through the computation

sequence. The user may limit the output in any desired manner by specifying from 1 to 30 curves in groups of one to three. After the output of each group, the program will print the command signal "\*\*\*" at which time the user may respond with:

- P or G continuing the output sequences as described above
- D initiating another run in which parameter changes may be made as described above.

Other commands which might be made are:

- N for NEW CASE which re-initiates the input cycle from the beginning. The program response to this is:

TYPE AIRCRAFT TYPE

allowing the user to load the file pertaining to another configuration or model of the same aircraft, or a different aircraft.

- L for LANDING which sets certain program switches pertinent to the landing calculation.

- E for END which stops execution.

Regardless of the number of steps  $N = T/\Delta T$ , the program stores only 50 values. All 50 values are used in graphing the variables shown in Table IV-2, but only 25 values are printed. If the user desires a more accurate resolution, he must accomplish this by decreasing the number of steps  $N = T/\Delta T$ , by shortening the run time  $T$ .  $N$  must equal at least 50.



Individual runs may be continuations of a prior run. Table IV-3 and Figure IV-3 illustrate the procedure. The values in Table IV-3 for case I are taken from Appendix A, Series II, case 1. The run time for this case was 15 seconds. The analyst chose to change thrust and fuselage trim at  $t = 9.6$  seconds. The initial conditions for case 1.1 are those which prevailed at  $t = 9.6$  in case 1, plus the control changes. This procedure may be repeated any number of times to compute the flight path of a maneuver, during each segment of which the controls are fixed.

TABLE JV-1  
PARAMETER CODES

AIRCRAFT INITIAL CONDITIONS

- |   |                    |
|---|--------------------|
| 1. A/C INERTIAL X POSITION              | (X)                |
| 2. A/C INERTIAL Z POSITION              | (Z)                |
| 3. A/C PITCH RATE                       | ( $\dot{\theta}$ ) |
| 4. A/C PITCH ATTITUDE                   | ( $\theta$ )       |
| 5. A/C VELOCITY - X DIRECTION-BODY AXES | (U)                |
| 6. A/C VELOCITY - Z DIRECTION-BODY AXES | (W)                |

AIRCRAFT CONTROLS

- |                              |                |
|------------------------------|----------------|
| 7. THRUST TO WEIGHT RATIO    | (T/W)          |
| 8. CYCLIC PITCH-LONGITUDINAL | ( $B_{1s}$ )   |
| 9. FUSELAGE PITCH TRIM ANGLE | ( $\theta_T$ ) |

SHIP MOTIONS

- |                                  |              |
|----------------------------------|--------------|
| 10. SHIP HEAVE AMPLITUDE         | ( $\eta_3$ ) |
| 11. SHIP PITCH AMPLITUDE         | ( $\eta_5$ ) |
| 12. SHIP ENCOUNTER FREQUENCY     | ( $\omega$ ) |
| 13. PHASE ANGLE - PITCH TO HEAVE | ( $\delta$ ) |

SHIP INITIAL CONDITIONS

- |                                    |  |
|------------------------------------|--|
| 14. INITIAL SHIP X CG INERTIAL     |  |
| 15. INITIAL SHIP Z CG INERTIAL     |  |
| 16. INITIAL LAUNCH/RECOVERY PT - X |  |
| 17. INITIAL LAUNCH/RECOVERY PT - Z |  |

TABLE IV-1 (Cont'd)

## SHIP VELOCITY

 $(V_s)$ WIND AND SEA STATE CONDITIONS

18. WIND VELOCITY

 $(V_H)$ 

19. SIGNIFICANT WAVE HEIGHT

 $(h_{1/3})$ 20. INTEGRATION CONSTANTS

21. RUN TIME

 $(T)$ 

22. TIME STEP

 $(\Delta T)$ PARAMETER CODESTHRUST TIME CONSTANTS

23. THRUST DELAY TIME

 $(D_D)$ 

24. THRUST BUILD-UP TIME

 $(D_B)$ 

## PROGRAM CODES

25. TIME AT INITIAL STEP

 $(T_0)$ 

26. CONTINUE

TABLE IV-2

OUTPUT PARAMETER LIST

CURVE NUMBER	CURVE LABEL	PARAMETER	
1	U A/C	A/C FLIGHT VELOCITY	BODY AXES
2	W A/C	A/C VERTICAL VELOCITY	BODY AXES
3	THETA DOT	A/C PITCH RATE	BODY AXES
4	THETA	A/C PITCH ATTITUDE	BODY AXES
5	X(I) A/C	A/C X POSITION	INERTIAL AXES
6	Z(I) A/C	A/C Z POSITION	INERTIAL AXES
7	XISCG	SHIP CG X POSITION	INERTIAL AXES
8	ZISCG	SHIP CG Z POSITION	INERTIAL AXES
9	FWD CP	NOSE WHEEL DECK CONTACT POINT	INERTIAL AXES
10	AFT CP	MAIN WHEEL DECK CONTACT POINT	INERTIAL AXES
11	FLGN	A/C NOSE GEAR LOAD	LBS
12	FLGM	A/C MAIN GEAR LOAD	LBS
13	B1F	CYCLIC PITCH-LONGITUDINAL	RAD
14	ALPHA	ROTOR ANGLE OF ATTACK	RAD
15	GAMMA	CLIMB (OR DESCENT) ANGLE	RAD
16	THRUST	THRUST	LBS
17	ZF	ROTOR Z FORCE (BODY AXES)	LBS
18	XF	ROTOR X FORCE (BODY AXES)	LBS
19	ZLG	LANDING GEAR FORCE	LBS
20	XLG	LANDING GEAR BRAKE FORCES	LBS
21	MLG	LANDING PITCHING MOMENT	FT-LBS

TABLE IV-2 (Cont'd)

CURVE NUMBER	CURVE LABEL	PARAMETER	
22	MGL	MAIN GEAR STROKE	INCHES
23	NGL	NOSE GEAR STROKE	INCHES
24	LF	LIFT RCTOR	LBS
25	DF	DRAG-ROTOR	LBS
26	LFUS	LIFT-FUSELAGE	LBS
27	DFUS	DRAG-FUSELAGE	LBS
28	LT	LIFT-TAIL	LBS
29	DT	DRAG-TAIL	LBS

TABLE IV-3

## FLIGHT PATH DATA - CH5 - SERIES II Cases 1 and 1.1

Initial Conditions  $t=0$   $x=z=\dot{x}=\dot{z}=0=0$   $T=33600$   $\theta_t=.014$ 

t	x	z	U ( $\dot{x}$ )	W ( $\dot{z}$ )	$\theta$	$\dot{\theta}$
0	0.0	-33.50	0.0	0.00	0.00	0.00
.6	-0.05	-34.45	-.43	-1.82	.00	0.00
1.2	-0.52	-35.55	-.62	-1.86	.01	0.013
. . .	. . .	. . .	. . .	. . .	. . .	. . .
9.0	5.68	-40.50	4.84	.81	-.019	-.0139
9.6	7.71	-39.91	2.06	.96	-.049	0.043
10.2	8.30	-39.24	1.21	1.18	.017	0.100
. . .	. . .	. . .	. . .	. . .	. . .	. . .
14.4	29.62	-35.21	6.43	-2.57	-.018	0.050
14.7	31.36	-35.94	5.51	-2.38	-.008	0.102

Initial Conditions @  $T = 9.6$  $x = 1.71$   $U = 2.06$   $\theta = -0.49$   $T = 33650$  $z = -39.91$   $W = .96$   $\dot{\theta} = 0.043$   $\theta_t = .044$ 

t	x	z	U ( $\dot{x}$ )	W ( $\dot{z}$ )	$\theta$	$\dot{\theta}$
9.6	7.71	-39.91	2.06	.96	-.049	0.043
10.2	11.61	-40.50	4.38	-1.90	-.055	-0.030
10.8	16.40	-42.16	6.95	-3.23	-.183	-0.045
. . .	. . .	. . .	. . .	. . .	. . .	. . .
16.2	25.30	-53.40	3.05	-3.87	-.238	-0.003
16.8	27.20	-55.37	3.43	-3.93	-.242	-0.008
17.4	28.96	-58.20	3.92	-4.16	-.256	+0.009
. . .	. . .	. . .	. . .	. . .	. . .	. . .
24.0	66.38	-77.30	8.93	-4.06	-.176	+0.072
24.4	70.24	-79.16	11.26	-3.92	-.154	+0.083

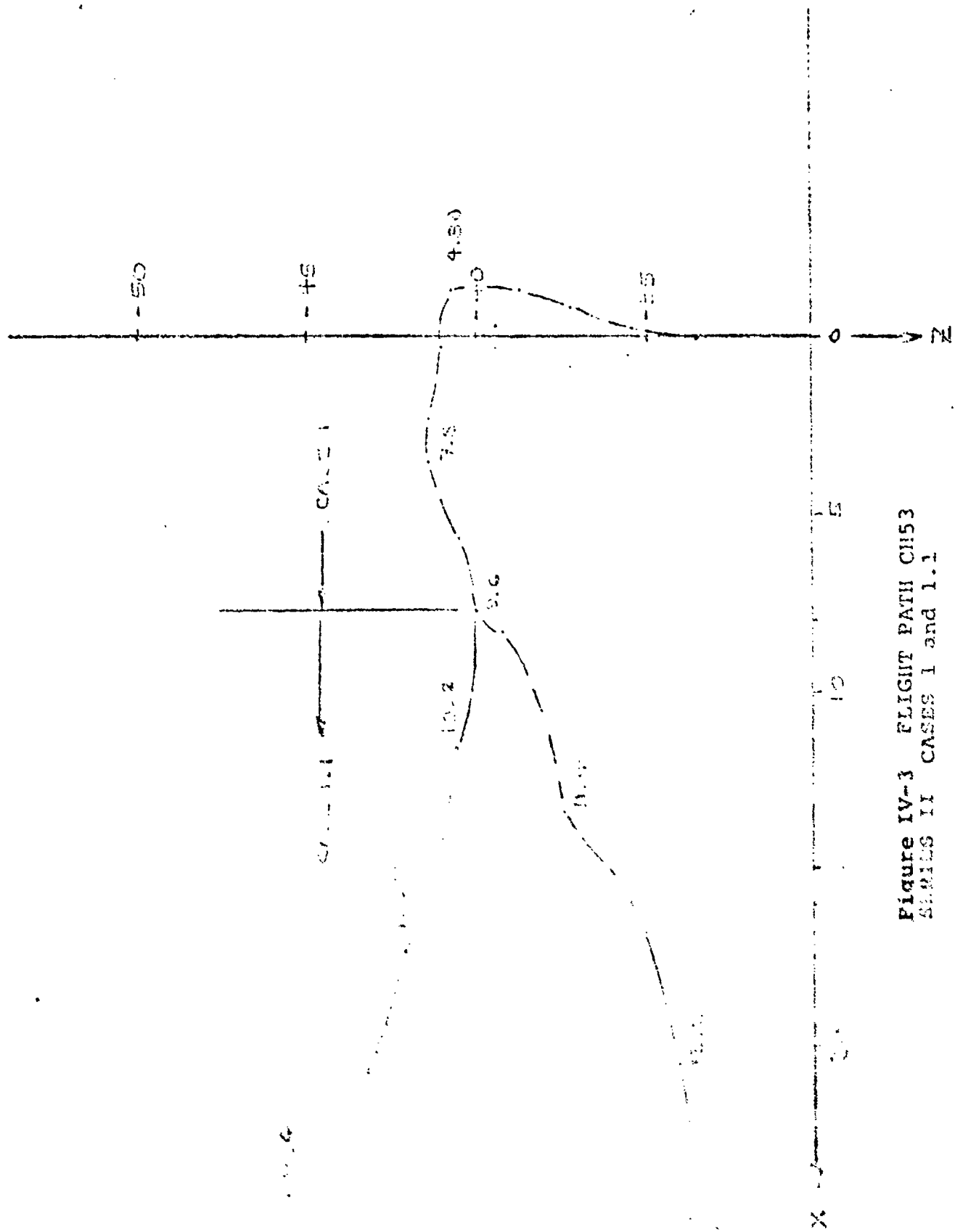


Figure IV-3 FLIGHT PATH CH53  
SERIES II CASES 1 and 1.1

## V. RESULTS

Over 350 computer runs were made, 34 of which are shown in the tabulations of Appendix A. The principal object of these runs was to test the program and to gain experience in its use. Thus, these runs do not necessarily represent a systematic evaluation of the aircraft/ship compatibility problem. Budget constraints prevented an in-depth parametric survey of all aspects of this problem. In fact, the "ship" itself is only an approximation, since no detailed and accurate information was available to model a specific ship design. However, important conclusions have been drawn about the effects of ship motions on aircraft control and stability:

(1) Even for the most severe ship motions (a wave height of 30 ft., an encounter frequency of 0.3 rad/sec., Series III, Case 11) the CH-53D control margin is adequate for normal take-off. For ship encounter frequencies from 0.3 to 0.68 rad/sec., corresponding roughly to periods of 2.05 to 9.0 seconds, the ship motions are quite slow in comparison to the aircraft motions. Series III, case 4, represents a severe ship motion. In this case, the ship is heaving in a 30-ft. wave with an amplitude of 18.3 feet, and a period of 9 seconds. The ship is at zero amplitude at  $t = 0$ . The maximum aircraft rotor thrust is attained at  $t = 5$  seconds and the aircraft is off the deck by  $t = 5.0$  seconds. Despite the large ship excursions, there is no



excessive build-up in landing gear forces. The aircraft pitches forward about 3.5 degrees during the first three seconds, lifting the main gear off the deck. The nose gear force remains low, however, and vanishes at about  $t = 5.0$  seconds. At no time during this 5-second period does the pitching acceleration of the aircraft exceed  $\pm 0.105$  rad/sec.

(2) In all the runs tabulated in Appendix III, take-off occurred from a rising deck, at a time when the deck upward velocity was near a maximum. Other runs, monitored visually, were made for take-off from descending decks, also at maximum velocity. In no case were large or destabilizing moments transmitted to the aircraft. Case 1 of Series III is typical. Here, the deck is ascending at a rate of  $1/2 h_{1/3} \dot{u}^2 = (15)(.68)^2 = 7.0$  ft/sec at  $t = 0$ . It is assumed that the aircraft is released at  $t = 0$  at a distance of 7.5 inches above the static gear extension. Even so, the maximum total gear load is only 46500 pounds at  $t = 0.6$  sec. (A/C gross weight is 33500 lbs.), and the landing gear pitching moment is -104000 in.-lbs. nose down. The pitching velocity (THETA DOT) remains very low.

(3) Since the ship pitch amplitude was small ( $5.492^\circ$  or less) all runs were made with take-off from a point above the ship center of gravity. In the worst case, the ship's pitching motion, leading the heave by as little as  $\frac{\pi}{2}$  radians, would add less than 1.3 feet to the total vertical excursion at either bow or stern.

(4) The gains for the AFCS pitch channel were determined for the CH-53D and agree with the optimum values calculated by Sikorsky (Ref. 5). These values are:

$$K_R = .4 \quad \text{rate given}$$

$$K_P = .8 \quad \text{proportional gain}$$

(5) In its present form, the program can be used to determine gains, stability margins or trim conditions, but the procedures are not "optimum". Experience has pointed out changes which can be made to effect rapid convergence to optimum values by use of adaptive methods. Once optimum gains and trim conditions are identified, stability margins can be readily identified by an analysis of the motion.

(6) Although roll and yaw are suppressed in this model, the simulation yields considerable data which provides insight into the roll behavior. The significant variations in  $\lambda$  (the inflow ratio) and  $B_{\chi_s}$  (the longitudinal cyclic) at frequencies in the neighborhood of 2 to 3 rad/sec., because of the effect on side force, could induce critical coupling for ship motions in the same frequency range. The coupling effects stem from the presence of  $\lambda$  in the side force equation and in the third harmonic flapping term  $b_1$ .

## REFERENCES

1. Salvesen, N. and W.E. Smith, "Comparison of Ship Motion Theory and Experiment for Mariner Hull and a Destroyer Hull with Bow Modification", Report 3337, NSRDC, June 1971.
2. Frank, W. and N. Salvesen, "The Frank Close-Fit Ship Motion Computer Program," Report 3289, NSRDC, 1970.
3. St. Denis, M. and W.N. Pierson, "On the Motion of Ships in Confused Seas", SNAME Trans., Vol. 61, 1953.
4. Garay, E.K. and E. Kisielowski, "Stability and Control of Compound Helicopters", USAVLABS Rept. 70-67, U.S. Army Air Mobility Research and Development Laboratory, Ft. Eustis, Va.
5. Cooper, D., "Stability and Control and Flying Qualities", Report SER-65111, Sikorsky Aircraft, 22 March 1963.
6. Anon., "Flight Manual", Model CH-53 A/O, NAVAIR 01-230HMA-1 15 April 1969, Department of the Navy.
7. Crisi, J., "Structural Analysis of Nose Landing Gear", Report SER-65031, Sikorsky Aircraft, 16 July 1964.
8. Crisi, J., "Structural Analysis of Main Landing Gear," Report SER-65030, Sikorsky Aircraft, 15 July 1964.
9. Anon., "Maintenance Instructions Manual Landing Gear, CH-53A and CH-53D Helicopters", NAVAIR 01-230HMA-2-2.2, 15 June 1970, Department of the Navy.
10. Carr, L.W., "CH-53A Wind Tunnel Study," Report SER-65110, Sikorsky Aircraft, 22 March 1963.
11. Carrigan, B., "CH-53A Wind Tunnel Study," Report SER-65202, Sikorsky Aircraft, 15 November 1963.
12. Gossow, A. and G. Meyers, "Aerodynamics of the Helicopter", MacMillan Co., 1952.
13. Bailey, F.J., "A Simplified Theoretical Method of Determining the Characteristics of a Lifting Rotor in Forward Flight", Report 716, National Advisory Committee on Aeronautics, 1941.
14. Anon., "Flow of Fluids through Valves, Fittings, and Pipes", Technical Paper 410, Crane Co., 1970.

## REFERENCES

15. Salvesen, N., E.O. Tuck, and O. Faltinsen, "Ship Motions and Sea Loads", SNAME paper no. 6, for presentation at the annual meeting, New York, N.Y., November 10, 1970.
16. Korvin-Kroubovsky, B.V., "Theory of Seakeeping", SNAME, 1961.
17. Tukey, J.W. and R.W. Hamming, "Measuring Noise Color", Bell Telephone Laboratories, Memorandum for File, MM-49-110-119, 1949.
18. Wiener, N., "Extrapolation, Interpolation and Smoothing of Stationary Time Series", Wiley, New York, 1949.
19. Moskowitz, L., W.J. Pierson and E. Mehr, "Wave Spectra Estimated from Wave Records by the OWS Weather Explorer and OWS Weather Reporter", New York University Technical Report, 1962, 1963.
20. Kinsman, Blair, "Wind Waves", Prentice-Hall, Inc., 1965.
21. Sverdrup, H.U., W.H. Munk, "Wind, Sea and Swell: Theory of Relations for Forecasting", U.S. Navy Hydrographic Office Publication 601, March 1947.
22. Weigel, R.L., "Oceanographic Engineering", Prentice-Hall, 1964.
23. Fields, A.S., "Digital Simulation of Sea Wave Height/Time Series", NSRDC Report 6-106/70, December 1970.

APPENDIX A  
SELECTED RESULTS OF  
COMPUTER RUNS

# PRINT-OUT LABELS AND UNITS

TIME	ELAPSED TIME	SECS.
U A/C	VELOCITY IN X DIRECTION	FT/SEC
W A/C	VELOCITY IN Z DIRECTION	FT/SEC
THETA DOT	PITCHING VELOCITY	RAD/SEC
THETA	PITCH ATTITUDE	RAD.
X(I) A/C	X POSITION, A/C C.G.	FT
Z(I) A/C	Z POSITION, A/C C.G.	FT
Z(I) S CG.	Z POSITION, SHIP C.G.	FT
Z FWD CP	Z POSITION FORWARD	FT
	LANDING GEAR	
	CONTACT POINT	
Z AFT CP	Z POSITION AFT	FT
	LANDING GEAR	
	CONTACT POINT	
NOSE GEAR	NOSE GEAR LOAD	LBS
MAIN GEAR	MAIN GEAR LOAD	LBS
ZLG	TOTAL LANDING GEAR LOAD	LBS
MGL	MAIN GEAR STRUT EXTENSION	INCHES
NGL	NOSE GEAR STRUT EXTENSION	INCHES
	(FULLY COMPRESSED=0 INCHES	
	FULLY EXTENDED=12 INCHES)	
ZF	ROTOR FORCE - Z DIRECTION	LBS
XF	ROTOR FORCE - X DIRECTION	LBS
DIS	CYCLIC PITCH	RAD
CANVA	FLIGHT PATH ANGLE	RAD

(NOTE: 25 functional values for each variable are stored of which only 23 values are printed in the following tables. See Page 17-19. Hence the extrema, (first two values in each column of the print-out tables) may or may not be included in the print-out (the histories.)

# SERIES I ZERO THRUST

CASE	SWH	T	AT	
1	0	15	.05	CD = 200
2	0	20	.05	CD = 100
3	10'	20	.05	
4	20'	20	.05	
5	30'	20	.05	
6	5'	60	.10	
7	10'	60	.10	
8	20'	60	.10	
9	50'	60	.10	
10	30	15	.05	DROP HEIGHT = 16"
11	30	15	.05	TAKE-OFF AREA 106' AFT OF MIDSHIPS

FOR ALL CASES (EXCEPT AS NOTED)

DROP HEIGHT = 7.5" (EXCEPT 10)

ENCOUNTER FREQUENCY = .18

TAKE-OFF AREA: MIDSHIPS (EXCEPT CASE 11)

T = Length of run seconds

AT = Time step, seconds

SWH = Significant wave height ( $h_{1/3}$ ), ft. (double amplitude)

TIME	" A/C	THETA DOT	THETA	GEAR LOAD	HGL	HGL	NGL
1.023	-1.253	-3.278	-3.211	-42133.320	1.810	1.810	2.791
14.009	1.023	3.272	3.212	-10575.135	7.502	7.502	7.502
2.023	2.023	2.233	2.023	-16575.135	7.502	7.502	7.502
3.023	-1.253	-2.278	2.023	-41195.893	1.852	1.852	3.426
4.023	1.253	2.272	-2.023	-33472.629	3.435	3.435	3.426
5.023	-2.023	-2.230	-2.023	-27322.701	3.975	3.975	3.426
6.023	2.023	2.225	2.023	-41279.211	2.023	2.023	3.426
7.023	3.426	2.226	-2.023	-32741.277	3.593	3.593	2.543
8.023	-3.426	-2.241	2.023	-32216.093	2.988	2.988	3.356
9.023	3.426	2.235	2.023	-35822.710	2.642	2.642	3.135
10.023	-2.133	-2.014	-2.023	-32119.520	3.508	3.508	2.936
11.023	2.023	2.212	2.023	-37942.323	2.331	2.331	2.426
12.023	3.426	2.220	-2.023	-31654.577	3.343	3.343	3.057
13.023	-2.023	-2.238	2.023	-31722.093	3.116	3.116	3.057
14.023	2.023	2.223	2.023	-36760.531	2.512	2.512	3.057
15.023	-2.023	-2.235	-2.023	-20933.797	2.629	2.629	2.936
16.023	2.023	2.220	2.023	-30552.734	2.412	2.412	2.426
17.023	3.426	2.234	-2.023	-32538.435	3.233	3.233	3.057
18.023	-2.023	-2.235	2.023	-33337.075	3.200	3.200	3.135
19.023	2.023	2.222	2.023	-37596.211	2.394	2.394	2.313
20.023	-2.023	-2.224	-2.023	-32225.730	3.610	3.610	2.313
21.023	2.023	2.237	2.023	-35444.477	2.564	2.564	3.336
22.023	3.426	2.229	-2.023	-33546.937	3.221	3.221	3.057
23.023	-2.023	-2.213	2.023	-33372.432	2.410	2.410	2.313
24.023	2.023	2.213	2.023	-32364.918	2.925	2.925	3.361
25.023	-2.023	-2.235	-2.023	-34282.312	3.565	3.565	2.313
26.023	2.023	2.226	2.023	-29322.065	2.732	2.732	2.313
27.023	-2.023	-2.224	-2.023		3.032	3.032	2.313
28.023	2.023	2.224	2.023				
29.023	-2.023	-2.224	-2.023				
30.023	2.023	2.224	2.023				

SERIES I CASE 1 ZERO THRUST ZERO WAVE HIGH DAMPING

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SERIES I CASE 2 ZERO THRUST ZERO WAVE  
LO DAMPING (Cont'd)

TIME	W A/C	THETA DEG	THETA	X(1) A/C	Z(1) A/C	Z(1) S CS
MINIMUM	-1.839	-0.270	-0.020	-0.200	-30.329	-3.352
MAXIMUM	1.164	0.270	0.020	0.200	-32.243	1.118
3.333	-3.500	0.022	0.020	0.022	-33.500	0.022
3.333	-1.300	-0.270	0.020	-0.022	-33.531	-0.474
3.333	-1.524	-0.254	0.020	-0.024	-33.775	-0.939
3.333	-0.312	0.019	0.027	-0.011	-34.174	-1.335
3.333	0.102	0.244	0.022	-0.021	-34.582	-1.633
3.333	-0.207	0.022	0.019	-0.022	-35.143	-2.100
3.333	-0.554	-0.310	0.020	-0.022	-35.469	-2.520
3.333	-0.007	-0.036	0.020	-0.020	-35.054	-2.920
3.333	-0.310	-0.022	0.021	-0.020	-35.026	-3.342
3.333	0.395	0.024	0.026	-0.024	-36.236	-3.228
3.333	0.274	0.024	0.019	-0.020	-36.254	-3.313
3.333	-0.190	-0.015	0.017	-0.001	-36.320	-3.350
3.333	-0.554	-0.039	0.021	-0.002	-36.234	-3.323
3.333	-0.121	-0.016	0.024	-0.000	-36.303	-3.223
3.333	0.605	0.026	0.019	-0.000	-35.557	-3.201
3.333	0.798	0.027	0.012	-0.000	-35.868	-2.937
3.333	0.312	-0.028	0.004	-0.001	-35.606	-2.556
3.333	-0.150	-0.030	0.005	-0.000	-35.324	-2.223
3.333	0.000	-0.027	0.000	-0.000	-34.054	-1.846
3.333	0.045	0.013	0.005	-0.000	-34.471	-1.431
3.333	1.112	0.031	-0.000	-0.000	-34.102	-0.980
3.333	0.679	0.022	-0.012	-0.000	-33.826	-0.525
3.333	0.005	-0.033	-0.012	-0.001	-33.320	-0.051
3.333	-0.025	-0.036	-0.000	-0.000	-32.775	0.424
3.333	0.605	0.037	-0.000	-0.000	-32.291	0.901
3.333	0.177	-0.020	-0.000	-0.000	-32.243	1.118

SERIES I CASE 3 ZERO THRUST 5' WAVE (Cont'd)

TIME	XLG	WGL	WGL	NOSE GEAR	WAIL GEAR	WLG
MINIMUM	3.200	2.367	2.000	3492.071	12300.321	-50302.291
MAXIMUM	10.000	3.400	7.500	3715.019	32202.041	57323.752
1.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
2.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
3.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
4.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
5.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
6.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
7.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
8.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
9.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
10.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
11.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
12.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
13.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
14.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
15.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
16.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
17.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
18.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
19.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
20.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
21.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
22.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
23.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
24.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
25.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
26.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
27.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
28.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
29.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
30.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
31.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
32.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
33.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
34.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
35.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
36.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
37.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
38.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
39.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
40.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
41.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
42.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
43.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
44.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
45.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
46.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
47.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
48.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
49.000	3.400	3.400	7.500	3715.019	32202.041	57323.752
50.000	3.400	3.400	7.500	3715.019	32202.041	57323.752

SERIES I CASE 3 ZERO THRUST 5' WAVE (Cont'd)

TIME	" A/C	THETA DOY	THETA	Z(I) A/C	Z(I) S CG	CEAL 10' WAVE
01.1830	-2.031	-2.274	-2.335	-39.442	-6.722	-31835.570
02.1830	1.622	0.200	3.354	-31.271	2.235	-14943.033
03.1830	-1.192	0.224	3.212	-33.523	2.220	-14642.009
04.1830	-2.031	-0.374	3.226	-33.927	-3.049	-27715.237
05.1830	-1.982	-0.338	3.241	-34.520	-1.373	-35113.431
06.1830	-3.722	1.029	3.245	-35.373	-2.771	-39935.573
07.1830	-0.629	3.243	3.242	-35.235	-3.637	-34423.632
08.1830	-0.037	1.215	3.243	-37.398	-4.373	-32727.316
09.1830	-1.131	-0.323	3.244	-37.732	-5.045	-32635.634
10.1830	-1.132	0.091	3.251	-38.191	-5.013	-34324.122
11.1830	-0.372	0.209	3.254	-38.622	-6.279	-37922.050
12.1830	0.239	0.225	3.242	-39.328	-0.417	-34344.231
13.1830	0.294	0.313	3.241	-39.346	-0.625	-34483.393
14.1830	-0.375	-0.320	3.233	-39.443	-6.732	-33131.145
15.1830	-0.435	-0.272	3.242	-39.314	-6.639	-3312.255
16.1830	0.232	-0.322	3.241	-39.122	-6.445	-37757.914
17.1830	0.291	0.223	3.232	-39.898	-6.121	-34745.122
18.1830	0.312	0.313	3.222	-39.027	-5.673	-32766.635
19.1830	0.425	-0.325	3.214	-38.115	-5.112	-32222.791
20.1830	0.193	-0.345	3.214	-37.422	-4.447	-32722.697
21.1830	0.733	-0.313	3.212	-36.655	-3.692	-37363.273
22.1830	1.534	0.225	3.222	-35.943	-2.863	-35725.336
23.1830	1.529	0.313	-2.011	-35.236	-1.976	-31435.355
24.1830	0.937	0.313	-0.210	-34.420	-1.049	-2924.269
25.1830	3.521	-0.245	-2.322	-33.448	-2.121	-31750.344
26.1830	3.775	-1.224	-0.322	-32.458	2.948	-37215.312
27.1830	1.516	3.321	-0.327	-31.595	1.781	-37124.322
28.1830	3.493	-0.341	-0.235	-31.271	2.235	-31242.316

SERIES I CASE 4 ZERO THRUST 10' WAVE



TIME	N A/C	THETA DOT	THETA	Z(1) A/C	Z(1) C CG
MINIMUM	-5.178	-0.373	-0.111	-49.078	-10.299
MAXIMUM	0.395	0.130	0.166	-29.264	6.135
0.000	-3.251	0.312	0.035	-33.520	2.320
0.001	-5.178	-0.373	0.085	-34.915	-2.592
1.000	-4.076	-0.053	0.110	-37.213	-5.132
2.000	-3.349	0.006	0.133	-39.216	-7.569
3.000	-2.594	0.000	0.142	-41.397	-9.951
4.000	-2.000	0.012	0.140	-43.435	-11.030
5.000	-2.054	-0.020	0.157	-45.133	-13.701
6.000	-2.257	-0.020	0.105	-46.775	-15.347
7.000	-1.103	0.014	0.106	-47.795	-16.624
8.000	-3.131	0.026	0.102	-48.779	-17.526
9.000	-3.374	0.014	0.152	-49.445	-19.295
10.000	0.259	0.002	0.144	-49.744	-18.209
11.000	0.632	0.004	0.133	-49.701	-18.135
12.000	1.515	0.014	0.113	-49.356	-17.024
13.000	1.998	0.014	0.072	-48.711	-16.713
14.000	2.265	0.004	0.079	-47.733	-15.496
15.000	2.433	-0.000	0.059	-46.430	-13.961
16.000	2.764	-0.009	0.039	-44.903	-12.145
17.000	3.074	-0.007	0.010	-43.263	-10.264
18.000	3.305	-0.000	-0.028	-41.274	-7.819
19.000	3.394	-0.013	-0.031	-38.922	-5.397
20.000	3.305	-0.015	-0.053	-30.645	-2.806
21.000	3.350	-0.017	-0.074	-34.295	-2.277
22.000	3.306	-0.010	-0.093	-31.029	2.317
23.000	3.238	-0.014	-0.111	-29.631	4.965
24.000	2.469	-0.025	-0.111	-22.264	7.125

SERIES I CASE 5 ZERO THRUST 15' WAVE

MINIMUM	TIME	NOSE GEAR	MAIN GEAR	WLS	GEAR LOAD	MGL	NGL	
	3.220	5483.681	3.220	-63136.344	-42556.945	1.931	2.846	
MAXIMUM	10.600	9300.270	33762.028	120077.025	-5463.001	13.492	7.568	
9.200		5483.981	3.320	120677.025	-5483.681	13.492	7.560	
9.800		5384.410	11821.203	34135.125	-23135.695	4.450	3.209	
1.600		8077.702	23087.125	-1902.944	-31154.820	2.616	3.549	
2.400		0149.032	31220.076	-56524.156	-39140.711	2.116	3.415	
3.200		8510.447	28612.052	-32354.522	-37121.122	2.422	3.147	
1.000		0876.342	25089.379	-5739.969	-34765.719	2.750	2.991	
4.000		9972.200	20620.477	-12042.944	-35482.734	2.619	3.254	
5.600		8744.412	31035.352	-52526.362	-42579.766	2.308	3.101	
6.400		9704.034	33762.028	-63136.344	-42556.945	1.931	3.132	
7.200		8000.001	30711.105	-33047.437	-39154.325	2.237	2.962	
9.200		0010.022	27570.005	-14870.156	-36507.523	2.585	2.947	
9.800		0034.012	27411.934	-15389.107	-36314.915	2.621	3.252	
9.000		5042.022	29140.733	-22420.252	-30990.025	2.530	3.148	
12.400		8034.055	21233.387	-16115.281	-36068.242	2.712	3.161	
11.000		8099.150	25153.068	-124.944	-34351.020	2.946	3.127	
12.000		0022.262	23026.437	11297.523	-32830.699	3.277	3.146	
12.000		3663.456	23220.480	14393.969	-31800.954	3.316	3.223	
13.600		8773.717	22842.855	13746.937	-31616.574	3.365	3.296	
14.480		0716.172	22392.344	15785.025	-31120.516	3.461	3.229	
15.200		8703.336	21559.141	21427.969	-32021.777	3.631	3.332	
15.200		9658.041	22700.719	26735.375	-29406.702	3.771	3.342	
15.800		9652.093	22424.445	20778.769	-29266.543	3.925	3.371	
17.000		6500.450	20291.107	20327.294	-28902.025	3.024	3.411	
16.400		6530.473	20185.215	27070.125	-29724.007	3.833	3.441	
13.200		8400.318	19912.633	23934.906	-28413.353	3.838	3.457	
10.600		9057.705	31119.434	-39171.531	-42177.141	2.143	2.000	

SERIES I CASE 5 ZERO THRUST 15' WAVE (Cont'd)



	TIME	2-FWD CP	2-REAR CP
MINIMUM	9.000	-40.220	-44.177
MAXIMUM	19.600	-19.430	-22.927
	0.000	-27.198	-25.253
	0.000	-22.967	-28.775
	1.000	-32.001	-31.244
	2.000	-35.223	-33.612
	3.000	-37.025	-35.330
	4.000	-39.750	-37.853
	4.800	-41.645	-39.055
	5.000	-43.227	-41.133
	6.000	-44.475	-42.433
	7.000	-45.302	-43.351
	8.000	-46.874	-43.333
	9.000	-46.022	-44.177
	9.600	-45.737	-44.264
	10.000	-45.187	-43.598
	11.000	-44.264	-42.789
	12.000	-42.686	-41.051
	12.000	-42.095	-40.225
	13.000	-38.983	-36.482
	14.000	-30.727	-30.515
	15.000	-31.206	-34.341
	16.000	-31.649	-32.117
	17.000	-28.331	-29.582
	18.000	-22.157	-27.364
	19.000	-23.414	-24.582
	19.600	-22.729	-22.123
	19.600	-19.430	-22.927

SERIES I CASE 5 ZERO THRUST 15' WAVE (Cont'd)

TIME	WAVE	2(1) A/C	2(1) S CG	2-FWD CP	2-REAR CP
1.000		-1.637	-1.503	-26.203	-26.251
1.001		1.506	1.592	-24.905	-24.347
2.000		-2.297	2.192	-26.550	-26.462
2.001		-1.577	-2.609	-27.264	-27.134
4.000		-2.269	-1.212	-27.632	-27.571
7.000		2.127	-1.594	-23.153	-27.922
9.000		-2.553	-1.572	-26.130	-26.337
12.000		1.546	-1.515	-27.077	-27.799
14.000		-1.219	-2.023	-27.331	-27.321
16.000		1.347	-2.175	-26.532	-26.601
19.000		-2.909	2.505	-25.928	-26.226
21.000		2.031	1.192	-25.232	-25.440
24.000		2.167	1.478	-24.932	-25.065
26.000		-2.313	1.593	-24.615	-24.847
28.000		2.019	1.413	-25.321	-25.115
31.000		-1.227	2.909	-25.517	-25.539
33.000		2.354	2.349	-26.192	-25.742
35.000		-1.616	-2.336	-26.913	-26.928
37.000		2.551	-2.250	-27.567	-27.419
39.000		-2.509	-1.413	-28.225	-27.622
41.000		2.349	-1.592	-28.322	-28.352
43.000		-2.874	-1.403	-28.362	-27.954
45.000		2.647	-1.192	-28.630	-27.592
47.000		-1.477	-2.517	-28.909	-27.224
49.000		1.233	2.192	-26.271	-26.367
51.000		1.151	2.312	-25.365	-25.725
53.000		-2.777	1.312	-25.268	-25.231
55.000		-2.923	1.475	-24.934	-25.200

SERIES I CASE 6 ZERO THRUST 2.5' WAVE

TIME	THETA DCI	THETA	MLG	NOSE GEAR	WING GEAR	GEAR LCAO
0.000	0.000	0.000	-90373.187	5493.971	12493.693	-45367.095
00.001	0.003	0.005	01774.097	0415.051	37139.133	-15077.572
0.002	0.001	0.003	35392.216	5493.973	12493.693	-15077.572
0.003	0.002	0.004	-27632.405	9105.096	28780.223	-24880.759
0.004	0.003	0.005	33351.150	8797.578	29790.359	-32262.247
0.005	0.004	0.006	-87833.125	9172.210	35973.539	-44191.117
0.006	0.005	0.007	47970.294	9511.405	10133.393	-20345.379
0.007	0.006	0.008	-29762.312	9631.443	28247.422	-30753.897
0.008	0.007	0.009	26337.713	9247.473	23340.277	-23527.052
0.009	0.008	0.010	28683.753	9242.335	21515.453	-22552.020
0.010	0.009	0.011	-42745.539	9415.361	23315.449	-37555.044
0.011	0.010	0.012	59443.037	9223.590	18233.371	-27624.332
0.012	0.011	0.013	-90373.137	9316.293	37130.120	-45367.095
0.013	0.012	0.014	61140.234	6300.242	17631.580	-20397.079
0.014	0.013	0.015	-57770.262	6062.235	31790.137	-43167.333
0.015	0.014	0.016	45417.156	6734.152	18031.033	-27263.009
0.016	0.015	0.017	1225.250	8179.238	24490.097	-32332.644
0.017	0.016	0.018	-124.553	9293.530	23782.977	-32190.215
0.018	0.017	0.019	47269.291	9101.187	19577.239	-29972.727
0.019	0.018	0.020	-83542.210	9240.976	34012.156	-43201.244
0.020	0.019	0.021	61774.687	9284.117	17677.941	-27224.477
0.021	0.020	0.022	-74561.562	9284.117	33332.649	-42216.727
0.022	0.021	0.023	50434.378	9257.273	18462.259	-27519.332
0.023	0.022	0.024	-12746.875	9222.209	25521.353	-34429.219
0.024	0.023	0.025	13112.344	9522.729	22223.758	-32746.463
0.025	0.024	0.026	39627.612	9252.381	23392.625	-29593.216
0.026	0.025	0.027	-63082.375	8212.777	32230.992	-41419.772
0.027	0.026	0.028	44375.844	8040.174	19353.098	-27952.274

SERIES I CASE 6 ZERO THRUST 2.5' WAVE (Cont'd)

	TIME	NGL	NGL
MINIMUM	2.220	1.679	2.641
MAXIMUM	55.301	9.337	7.500
1	0.330	8.337	7.560
3	2.400	2.367	3.659
5	4.000	3.579	2.035
7	7.230	1.750	3.472
9	9.000	3.974	2.893
11	12.000	2.512	3.221
13	14.400	3.383	3.399
15	16.900	3.695	3.672
17	19.200	2.215	3.626
19	21.500	4.320	2.541
21	24.000	1.679	3.613
23	26.300	4.421	2.795
25	29.799	2.132	3.392
27	31.199	3.020	3.162
29	33.000	3.162	3.210
31	36.330	2.834	3.543
33	33.400	4.353	2.721
35	40.300	1.813	3.676
37	43.200	4.436	2.723
39	45.600	1.556	3.524
41	49.300	4.304	3.263
43	53.401	2.347	3.114
45	52.301	3.284	3.473
47	55.201	3.037	2.742
49	57.601	1.983	3.649
51	59.901	3.871	3.172

SERIES I CASE 6 ZERO THRUST 2.5' WAVE (Cont'd)

TIME	X F/C	Z(1) F/C	Z(1) S CG	Z F/C S CG	2-8-55 2-8-55
11:25	-1.287	-35.247	-3.189	-3.189	-23.035
11:26	1.092	-33.197	3.181	3.181	-23.035
11:27	-0.575	-33.582	0.322	0.322	-23.035
11:28	-1.212	-34.127	-1.237	-1.237	-23.035
11:29	-1.509	-35.148	-2.426	-2.426	-23.035
11:30	-1.144	-35.762	-3.168	-3.168	-23.035
11:31	-0.579	-36.115	-3.144	-3.144	-23.035
11:32	1.412	-35.592	-2.630	-2.630	-23.035
11:33	-1.791	-34.742	-1.647	-1.647	-23.035
11:34	1.860	-33.697	-3.351	-3.351	-23.035
11:35	-0.422	-32.143	1.112	1.112	-23.035
11:36	-0.771	-31.382	2.121	2.121	-23.035
11:37	1.526	-32.237	2.955	2.955	-23.035
11:38	-1.482	-32.357	3.191	3.191	-23.035
11:39	1.334	-33.346	2.319	2.319	-23.035
11:40	-1.474	-31.327	1.937	1.937	-23.035
11:41	0.654	-32.366	3.637	3.637	-23.035
11:42	-1.007	-33.362	-3.671	-3.671	-23.035
11:43	-1.234	-34.352	-1.916	-1.916	-23.035
11:44	-0.631	-33.400	-2.617	-2.617	-23.035
11:45	-1.115	-36.247	-3.179	-3.179	-23.035
11:46	-1.615	-35.775	-2.965	-2.965	-23.035
11:47	1.692	-35.356	-2.213	-2.213	-23.035
11:48	-1.832	-34.222	-1.335	-1.335	-23.035
11:49	1.236	-32.693	0.325	0.325	-23.035
11:50	-1.236	-31.379	1.524	1.524	-23.035
11:51	-0.335	-32.552	2.624	2.624	-23.035
11:52	-0.951	-33.513	2.951	2.951	-23.035

SERIES I CASE 7 ZERO THRUST 5' WAVE

TIME	NOSE GEAR	MAIN GEAR	GEAR LOAD	THETA DOI	THETA	ALG
MINIMUM	5480.071	9000.264	-45200.572	-3.291	-0.230	-00477.025
MAXIMUM	5410.141	35000.326	-15442.130	2.103	0.238	52112.734
2.0000	5423.071	3950.204	-15442.130	3.222	0.216	35197.302
2.0000	5432.340	20310.331	-34979.383	-3.251	0.223	-20235.344
4.0000	5401.035	21791.897	-32870.523	-0.204	0.201	27070.000
7.0000	5421.570	34773.773	-43152.291	0.312	0.225	-75545.162
9.0000	5415.402	21325.072	-32421.164	-0.240	0.217	31405.000
12.0000	5455.253	27792.034	-30447.077	0.300	0.210	-20009.000
14.0000	5415.077	22000.000	-31300.000	-0.277	0.204	17000.000
15.0000	5410.270	22500.227	-23751.400	0.252	-0.210	2077.757
19.0000	5417.023	32000.777	-43314.500	-0.252	-0.203	-02000.000
21.0000	5410.141	17034.754	-27253.005	0.310	-0.200	52112.734
24.0000	5427.244	30000.343	-45200.572	3.221	-0.200	-00477.025
26.0000	5423.010	17733.000	-27000.000	-0.231	-0.232	02314.000
28.0000	5412.045	22000.281	-30310.000	0.271	-0.200	-50040.437
31.0000	5420.014	10130.102	-28161.410	-0.263	-0.200	42341.437
33.0000	5420.422	21113.320	-32705.750	0.278	0.215	50040.000
35.0000	5406.407	24472.004	-30000.422	-0.272	0.222	-50040.000
37.0000	5417.919	10200.487	-20000.301	0.212	0.214	10770.000
40.0000	5417.187	30000.001	-40000.149	-0.217	0.200	-00000.000
43.0000	5420.030	17727.211	-27027.227	0.217	0.213	02491.227
45.0000	5410.293	32241.494	-42554.781	-0.254	0.225	-62000.000
48.0000	5400.248	15000.422	-27011.672	0.263	0.223	4251.000
51.0000	5410.753	24000.174	-30000.000	-0.260	-0.225	0011.710
54.0000	5400.000	24044.682	-32641.586	-0.201	-0.200	-3714.252
55.0000	5422.011	10445.920	-28700.000	0.242	-0.231	40707.731
57.0000	5410.701	34922.723	-43130.404	-0.233	-0.213	-92017.150
58.0000	5411.554	10638.000	-20440.000	-0.272	-0.200	20000.000

SERIES I CASE 7 ZERO THRUST 5' WAVE (Cont'd)

TIME	W A/C	Z(1) A/C	Z(1) S CG	INELA DEL	INELA	SLC
0.000	2.576	-44.327	-12.181	0.000	-2.123	-51.875.000
50.001	2.525	-22.244	12.194	0.004	0.129	50.87.077
0.002	2.521	-33.522	0.022	0.026	0.023	50.87.017
0.003	2.570	-37.220	-5.119	-0.020	0.032	-50.87.000
0.004	2.570	-41.072	-0.004	0.020	0.120	0.000.000
7.000	1.411	-40.000	-11.752	0.115	0.120	-4.980.125
9.000	2.420	-44.100	-12.042	-0.125	0.030	-0.000.120
12.000	1.412	-42.025	-12.120	0.010	0.052	-12.23.007
14.000	2.420	-39.310	-6.300	2.020	0.120	-1.400.150
16.000	2.430	-34.007	-1.244	0.112	-0.239	0.771.710
18.000	2.414	-31.120	0.000	0.112	-0.077	1.232.721
21.000	2.257	-21.910	0.007	0.117	-0.122	1.000.075
24.000	1.476	-20.040	11.520	0.121	-0.127	219.13.522
26.000	1.570	-22.144	12.184	0.026	0.024	25722.097
28.000	1.521	-23.379	13.700	0.032	-0.163	20000.075
31.000	-1.343	-25.000	7.420	0.032	-0.221	27502.701
33.000	-1.521	-31.031	2.671	0.028	0.024	20222.031
36.000	-1.709	-35.021	-2.571	0.019	0.065	20070.531
39.000	-1.634	-30.459	-7.033	0.025	0.093	20005.210
41.000	2.827	-44.225	-12.179	-0.014	0.124	20002.150
43.000	2.159	-43.061	-11.358	-0.031	0.000	22302.504
45.000	2.510	-41.102	-3.443	-0.044	0.070	10002.044
48.000	1.300	-37.212	-3.004	-0.043	0.000	700.044
52.000	1.351	-32.470	1.244	-0.045	-0.012	-12477.450
55.000	1.330	-27.974	0.220	-0.036	-0.054	-26270.156
58.000	2.000	-24.200	12.052	0.022	-0.035	-30000.000
60.001	1.324	-23.201	11.033	-0.027	-0.122	-41661.437
60.001				0.023	-0.120	31122.437

SERIES I CASE 8 ZERO THRUST 10' WAVE

MINIMUM	TIME	Z-FWD CP	Z-REAR CP	NOSE GEAR	MAIN GEAR	GEAR LOAD
	0.000	-30.556	-30.312	5497.87	7504.209	-41116.477
MAXIMUM	50.801	-13.528	-14.705	9147.739	32200.504	-12939.350
	0.000	-26.905	-26.335	5403.071	7524.209	-12000.000
	2.400	-32.437	-31.327	9103.323	33316.663	-30524.000
	4.000	-35.745	-35.414	9033.227	25553.676	-34493.883
	7.200	-39.135	-37.659	9015.973	32200.524	-41116.477
	0.000	-39.311	-39.217	9232.564	33007.145	-39097.711
	12.000	-37.307	-36.417	9147.739	33442.420	-37501.219
	14.400	-32.003	-32.703	9207.532	27232.730	-26222.570
	16.000	-27.495	-27.906	9205.532	24970.793	-35075.625
	19.200	-21.947	-22.900	9004.350	23941.272	-32035.617
	21.000	-17.252	-18.541	9052.661	22531.905	-31484.828
	24.000	-14.260	-15.024	9001.971	21034.100	-32730.000
	26.400	-13.628	-14.705	8625.000	21297.530	-33122.072
	28.000	-15.178	-15.057	9275.122	21127.070	-29992.102
	31.200	-16.921	-19.157	9222.912	22745.300	-29468.379
	33.000	-24.205	-23.715	8637.670	21329.200	-28936.931
	36.000	-29.051	-29.700	8437.902	19644.531	-28092.523
	39.400	-34.641	-33.410	9249.408	19153.129	-27309.530
	42.000	-38.124	-36.759	9157.902	19232.695	-27392.578
	43.200	-39.409	-38.222	8177.212	19941.101	-29119.426
	45.600	-39.427	-37.532	9205.209	21429.789	-29695.278
	49.200	-35.184	-34.912	9416.020	23076.523	-32300.330
	52.400	-32.334	-30.536	9604.867	20211.129	-34810.000
	52.600	-24.790	-25.530	9207.299	20502.441	-27359.742
	55.200	-19.557	-23.717	9071.040	32434.007	-39426.711
	57.000	-15.625	-16.379	9117.500	31639.002	-43757.331
	59.800	-14.377	-15.736	9495.961	19570.541	-28264.000

SERIES I CASE 8 ZERO THRUST 10' WAVE (Cont'd)



TIME	HCL	HCL	HCL
MINIMUM	2.222	2.224	2.222
MAXIMUM	53.301	11.514	7.552
0.222	11.514		7.552
2.422	2.121		2.227
4.322	2.737		2.222
7.222	2.251		2.223
9.522	2.212		2.222
12.222	2.220		2.222
14.222	2.283		2.224
16.222	2.227		2.222
18.222	3.225		2.222
21.222	3.253		2.122
24.222	3.542		2.212
26.422	3.654		2.225
29.322	3.670		2.227
31.222	3.747		2.222
33.622	3.819		2.222
36.222	3.245		2.222
38.422	4.223		2.222
41.222	3.222		2.222
43.222	3.222		2.222
45.222	3.222		2.222
48.222	3.222		2.222
51.222	3.222		2.222
54.222	3.222		2.222
57.222	3.222		2.222
59.222	3.222		2.222

SERIES I CASE 8 ZERO THRUST 10' WAVE (Cont'd)

	Z(1) A/C	Z(1) S CC	Z-FWD CP	Z-REAR CP
1000	-5.523	-32.454	-56.565	-55.456
1001	5.521	32.402	4.853	1.913
1002	5.523	3.232	-27.664	-26.398
1003	-3.529	-12.793	-41.125	-38.426
1004	-1.527	-23.235	-51.097	-48.371
1005	-0.523	-28.031	-57.040	-54.329
1006	1.529	-32.124	-59.216	-55.292
1007	3.529	-25.269	-52.345	-51.223
1008	5.527	-15.772	-42.391	-42.175
1009	7.527	-3.353	-23.189	-32.422
1010	9.527	9.671	-15.648	-18.132
1011	11.527	22.317	-4.327	-7.529
1012	13.527	28.251	2.015	-2.491
1013	15.527	33.482	4.853	1.912
1014	17.527	26.997	1.116	-2.823
1015	19.527	13.552	-7.749	-9.329
1016	21.527	0.676	-26.117	-19.242
1017	23.527	-8.426	-33.552	-31.372
1018	25.527	-19.375	-45.472	-42.553
1019	27.527	-28.379	-53.012	-52.393
1020	29.527	-33.446	-57.224	-54.113
1021	31.527	-39.394	-55.127	-52.665
1022	33.527	-41.399	-47.713	-48.757
1023	35.527	-5.929	-36.046	-36.852
1024	37.527	3.111	-23.228	-25.124
1025	39.527	15.050	-12.511	-13.312
1026	41.527	25.132	-1.509	-4.985
1027	43.527	29.255	1.422	-1.975

SERIES I CP 9 ZERO THRUST 25° WAVE





Wavelength	Frequency	U A C	φ A/C	Theta DOT	Theta	X(1) A/C	Z(1) A/C
Wavelength	Frequency	U A C	φ A/C	Theta DOT	Theta	X(1) A/C	Z(1) A/C
Wavelength	Frequency	U A C	φ A/C	Theta DOT	Theta	X(1) A/C	Z(1) A/C
14.700	20400	0.000	4.295	-0.000	-0.000	-0.953	-49.890
14.700	20400	0.015	2.197	0.115	0.157	0.072	-34.000
14.700	20400	0.030	-3.163	0.012	-0.006	0.000	-34.000
14.700	20400	0.045	-4.295	-0.000	0.000	-0.000	-34.597
14.700	20400	0.060	-1.340	0.115	0.000	-0.000	-36.971
14.700	20400	0.075	-4.295	-0.000	0.000	-0.000	-33.448
14.700	20400	0.090	-1.541	0.091	0.000	-0.000	-39.927
14.700	20400	0.105	-2.408	0.017	0.000	-0.000	-41.663
14.700	20400	0.120	-2.935	-0.000	0.000	-0.000	-42.831
14.700	20400	0.135	1.477	0.000	0.000	-0.000	-44.000
14.700	20400	0.150	-2.935	-0.000	0.000	-0.000	-45.537
14.700	20400	0.165	-1.110	0.000	0.000	-0.000	-46.500
14.700	20400	0.180	-1.570	0.000	0.000	-0.000	-47.644
14.700	20400	0.195	-1.467	-0.000	0.000	-0.000	-48.219
14.700	20400	0.210	-2.408	0.000	0.000	-0.000	-49.000
14.700	20400	0.225	-1.000	-0.000	0.000	-0.000	-49.401
14.700	20400	0.240	0.000	0.000	0.000	-0.000	-49.645
14.700	20400	0.255	-0.000	0.000	0.000	-0.000	-49.890
14.700	20400	0.270	0.000	0.000	0.000	-0.000	-49.626
14.700	20400	0.285	0.000	0.000	0.000	-0.000	-49.575
14.700	20400	0.300	0.000	-0.000	0.000	-0.000	-49.000
14.700	20400	0.315	0.000	0.000	0.000	-0.000	-48.491
14.700	20400	0.330	0.000	0.000	0.000	-0.000	-47.806
14.700	20400	0.345	0.000	0.000	0.000	-0.000	-46.741
14.700	20400	0.360	0.000	-0.000	0.000	-0.000	-45.894
14.700	20400	0.375	0.000	0.000	0.000	-0.000	-44.532
14.700	20400	0.390	0.000	0.000	0.000	-0.000	-43.310
14.700	20400	0.405	0.000	-0.000	0.000	0.000	-42.491

SERIES I CASE 10

	TIME	MGL	MGL	NGL	NOSE GEAR	MAIN GEAR
MINIMUM	3-000	3-912	2-699	3-000	3-000	3-000
MAXIMUM	14-700	12-000	12-000	9626-010	52716-947	
	3-000	12-000	12-000	3-000	3-000	3-000
	3-000	3-912	3-490	8673-461	52716-947	
	1-200	3-175	2-737	9050-164	17294-270	
	1-400	3-929	3-434	9635-064	17327-100	
	2-400	2-100	3-410	8321-979	32736-337	
	3-000	4-207	2-839	9234-348	18493-896	
	3-500	2-511	3-693	8164-215	29027-871	
	4-200	3-304	3-049	8807-954	22539-091	
	4-400	3-330	3-315	8654-854	21902-922	
	5-400	2-497	3-000	8311-500	29268-892	
	6-000	3-910	3-040	8905-893	19331-099	
	6-000	2-400	3-041	8176-520	28711-602	
	7-200	3-404	3-129	8751-893	21719-941	
	7-400	3-096	3-456	8467-892	23408-893	
	8-400	2-700	3-473	8342-111	20108-641	
	9-000	3-700	3-106	8777-318	20398-336	
	9-400	2-301	3-009	8194-391	23280-426	
	10-700	3-041	3-006	9835-229	20398-797	
	12-000	2-000	3-000	8330-000	25329-801	
	11-000	3-000	3-000	8463-700	24158-936	
	12-000	3-000	3-000	8742-000	21276-332	
	12-000	2-000	3-000	8261-311	29736-969	
	13-000	3-700	3-000	8918-662	20251-531	
	14-000	2-000	3-000	8314-010	26964-286	
	14-000	3-144	3-100	8699-592	23719-112	
	14-700	2-300	3-000	8297-000	23384-537	

SERIES I CASE 10 (Cont'd)

MINIMUM	TIME	ZERO S CG	Z-FWD CP	Z-REAR CP
	2-000	-19-296	-46-139	-46-230
MAXIMUM	14-700	2-000	-26-399	-26-633
3-000	3-000		-26-399	-26-557
3-000	3-000	-1-999	-29-424	-29-375
1-200	1-200	-3-257	-30-438	-30-193
1-000	1-000	-5-545	-32-415	-31-071
2-400	2-400	-7-354	-34-318	-33-097
3-000	3-000	-9-244	-36-140	-35-295
3-000	3-000	-12-039	-37-898	-36-370
4-200	4-200	-12-118	-39-453	-39-321
4-000	4-000	-13-469	-42-909	-40-624
5-400	5-400	-14-675	-42-210	-42-737
6-000	6-000	-15-725	-43-341	-41-775
6-800	6-800	-16-627	-44-294	-42-623
7-200	7-200	-17-312	-45-055	-43-296
7-800	7-800	-17-331	-45-621	-43-792
8-600	8-600	-19-161	-45-994	-44-105
9-000	9-000	-18-293	-46-139	-44-230
9-800	9-800	-19-256	-46-388	44-171
10-000	10-000	-17-301	-45-827	-43-921
10-800	10-800	-17-535	-45-559	-43-490
11-400	11-400	-16-901	-44-700	-42-877
12-000	12-000	-16-397	43-842	-42-090
12-600	12-600	-15-100	-42-901	-41-139
13-200	13-200	-13-953	41-593	-40-329
13-800	13-800	-12-656	-40-225	-39-774
14-400	14-400	-11-225	-39-679	-38-386
14-700	14-700	-10-453	-37-865	-36-645

SERIES I CASE 10 (Cont'd)

TIME	WAVE	WAVE	THETA DOT	THETA	X(1) A/C	Z(1) A/C
0.000	0.000	-2.039	-2.039	-2.039	-100.000	-42.042
1.000	0.000	1.000	0.000	0.000	-99.985	-33.500
2.000	0.000	-1.017	0.012	-0.006	-100.000	-33.000
3.000	0.000	-2.004	0.004	-0.006	-99.991	-34.721
4.000	0.000	-1.100	0.005	-0.017	-100.000	-35.942
5.000	0.000	-1.905	0.009	-0.034	-100.000	-36.764
6.000	0.000	-1.001	0.012	-0.052	-100.000	-37.582
7.000	0.000	-1.021	0.037	-0.068	-100.000	-38.433
8.000	0.000	-2.022	0.040	-0.076	-100.000	-39.333
9.000	0.000	-1.001	0.035	-0.039	-100.000	-40.000
10.000	0.000	-1.000	0.009	-0.105	-100.000	-40.000
11.000	0.000	-2.027	0.036	-0.114	-100.000	-41.000
12.000	0.000	-1.000	0.010	-0.121	-100.000	-41.000
13.000	0.000	-1.001	0.003	-0.131	-100.000	-42.000
14.000	0.000	-2.027	0.007	-0.140	-100.000	-42.000
15.000	0.000	-1.000	0.024	-0.144	-100.000	-42.653
16.000	0.000	-2.022	0.009	-0.140	-100.000	-42.840
17.000	0.000	-1.000	0.019	-0.140	-100.000	-42.000
18.000	0.000	-2.012	0.004	-0.150	-100.000	-42.000
19.000	0.000	-1.012	0.014	-0.148	-100.000	-42.490
20.000	0.000	-2.015	0.012	-0.145	-100.000	-42.000
21.000	0.000	-1.015	0.022	-0.144	-100.000	-41.000
22.000	0.000	-1.015	0.003	-0.139	-100.000	-41.000
23.000	0.000	-2.010	0.000	-0.130	-100.000	-40.000
24.000	0.000	-1.010	0.003	-0.120	-100.000	-40.000
25.000	0.000	-2.003	0.003	-0.114	-100.000	-39.000
26.000	0.000	-1.003	0.004	-0.103	-99.992	-38.472
27.000	0.000	-2.001	0.009	-0.096	-99.985	-38.000

SERIES I CASE 11



ITEM	Z-11 CP	Z-14 CP	Z-16 CP	NOSE GEAR	MAIN GEAR	ZLG
1-100	-10.200	-30.121	-37.219	9197.735	16168.341	-125234.87
1-100	2.000	-26.044	-27.295	12631.736	112653.141	-24365.355
1-100	3.000	-20.314	-27.295	12631.736	112653.141	-125234.87
1-100	-1.000	-24.212	-28.200	9456.932	33367.703	-40354.539
1-100	-3.000	-27.033	-29.217	9709.414	21132.414	-29341.828
1-100	-5.000	-30.050	-30.314	9351.055	21290.955	-29611.910
1-100	-7.000	-33.067	-31.560	9000.816	25773.992	-35085.812
1-100	-9.000	-36.084	-32.323	8933.203	28132.660	-37115.867
1-100	-11.000	-39.101	-33.222	8834.113	22077.091	-30611.835
1-100	-13.000	-42.118	-34.219	8407.810	21294.347	-29691.897
1-100	-15.000	-45.135	-34.794	8761.125	27077.703	-36463.929
1-100	-17.000	-48.152	-35.450	8546.893	26439.199	-35265.991
1-100	-19.000	-51.169	-36.210	8424.355	21304.129	-29783.484
1-100	-21.000	-54.186	-36.469	8372.393	22592.148	-30964.243
1-100	-23.000	-57.203	-36.822	8929.367	27739.343	-36618.914
1-100	-25.000	-60.220	-37.004	8723.268	25620.273	-34349.539
1-100	-27.000	-63.237	-37.194	8323.059	20950.316	-29273.375
1-100	-29.000	-66.254	-37.213	9424.549	22819.629	-31244.130
1-100	-31.000	-69.271	-37.118	9311.475	23705.547	-37619.023
1-100	-33.000	-72.288	-36.909	9665.205	23762.680	-32417.837
1-100	-35.000	-75.305	-35.591	8299.660	21103.937	-29433.593
1-100	-37.000	-78.322	-36.167	8526.467	24775.052	-33300.117
1-100	-39.000	-81.339	-35.639	8977.229	27923.215	-36700.445
1-100	-41.000	-84.356	-35.312	8571.053	23307.316	-31894.371
1-100	-43.000	-87.373	-34.295	8379.406	21259.539	-29637.945
1-100	-45.000	-90.390	-34.496	8672.221	20850.164	-34522.383
1-100	-47.000	-93.407	-32.620	8901.912	27000.480	-36607.391
1-100	-49.000	-96.424	-32.156	8509.012	23437.223	-31945.715

SERIES I CASE 11 (Cont'd)

MINIMUM	TIME	NGL	NGL	NGL
MINIMUM	0.000	0.000	0.000	0.000
MAXIMUM	14.000	4.000	4.000	4.000
	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000
	1.000	0.000	0.000	0.000
	1.000	0.000	0.000	0.000
	2.000	0.000	0.000	0.000
	3.000	0.000	0.000	0.000
	3.000	0.000	0.000	0.000
	4.000	0.000	0.000	0.000
	4.000	0.000	0.000	0.000
	5.000	0.000	0.000	0.000
	6.000	0.000	0.000	0.000
	6.000	0.000	0.000	0.000
	7.000	0.000	0.000	0.000
	7.000	0.000	0.000	0.000
	8.000	0.000	0.000	0.000
	9.000	0.000	0.000	0.000
	9.000	0.000	0.000	0.000
	10.000	0.000	0.000	0.000
	10.000	0.000	0.000	0.000
	11.000	0.000	0.000	0.000
	11.000	0.000	0.000	0.000
	12.000	0.000	0.000	0.000
	12.000	0.000	0.000	0.000
	13.000	0.000	0.000	0.000
	13.000	0.000	0.000	0.000
	14.000	0.000	0.000	0.000
	14.000	0.000	0.000	0.000

SERIES I CASE 11 (Cont'd)

SERIES II NO WAVE MOTION

CASE	THRUST	$\theta_t$	$K_R$	$K_P$
1	33600	-.014	.4	.8
2	33650	-.014	.4	.8
3	33650	-.0223	.2	.4
4	33700	-.035	.4	.8
5	33750	-.032	.4	.8
6	33750	-.0215	.4	.8
9	33800	-.0275	.4	.8
10	33800	-.0198	.4	.8
11	33800	-.0205	.4	.8
14	33850	-.0190	.4	.8
15	33850	-.0192	.4	.6
16	33850	-.0200	.3	.6

TIME	U A/C	THETA DOT	PHETA	X(1) A/C	Z(1) A/C
0.000	-2.231	-0.145	-0.052	-1.377	-21.454
0.001	0.000	0.000	0.000	31.360	-33.500
0.002	0.000	0.000	0.000	0.000	-33.500
0.003	-1.127	0.000	-0.004	-0.003	-34.449
0.004	-2.000	0.000	0.000	-0.004	-35.554
0.005	0.000	0.000	-0.012	-0.007	-36.659
0.006	-2.004	0.000	0.004	-0.007	-37.720
0.007	1.141	-0.001	0.016	-1.003	-38.669
0.008	0.000	-0.001	-0.007	-0.007	-39.420
0.009	-2.000	0.000	-0.017	-0.007	-40.109
0.010	1.141	-0.001	0.000	-1.003	-40.695
0.011	0.000	-0.001	-0.004	0.000	-41.000
0.012	-2.004	0.000	0.000	0.000	-41.282
0.013	1.141	-0.001	-0.007	0.000	-41.454
0.014	0.000	-0.001	0.000	0.000	-41.393
0.015	-2.000	0.000	-0.012	0.000	-41.215
0.016	1.141	-0.001	0.000	0.000	-40.974
0.017	0.000	-0.001	-0.007	0.000	-40.600
0.018	-2.004	0.000	0.000	0.000	-39.909
0.019	1.141	-0.001	-0.017	0.000	-39.243
0.020	0.000	-0.001	0.000	0.000	-38.376
0.021	-2.000	0.000	-0.007	0.000	-37.358
0.022	1.141	-0.001	0.000	0.000	-36.260
0.023	0.000	-0.001	-0.012	0.000	-35.000
0.024	-2.004	0.000	0.000	0.000	-34.000
0.025	1.141	-0.001	-0.017	0.000	-33.710
0.026	0.000	-0.001	0.000	0.000	-33.000
0.027	-2.000	0.000	-0.007	0.000	-32.000
0.028	1.141	-0.001	0.000	0.000	-31.000
0.029	0.000	-0.001	-0.012	0.000	-30.000
0.030	-2.004	0.000	0.000	0.000	-29.000
0.031	1.141	-0.001	-0.017	0.000	-28.000
0.032	0.000	-0.001	0.000	0.000	-27.000
0.033	-2.000	0.000	-0.007	0.000	-26.000
0.034	1.141	-0.001	0.000	0.000	-25.000
0.035	0.000	-0.001	-0.012	0.000	-24.000
0.036	-2.004	0.000	0.000	0.000	-23.000
0.037	1.141	-0.001	-0.017	0.000	-22.000
0.038	0.000	-0.001	0.000	0.000	-21.000
0.039	-2.000	0.000	-0.007	0.000	-20.000
0.040	1.141	-0.001	0.000	0.000	-19.000
0.041	0.000	-0.001	-0.012	0.000	-18.000
0.042	-2.004	0.000	0.000	0.000	-17.000
0.043	1.141	-0.001	-0.017	0.000	-16.000
0.044	0.000	-0.001	0.000	0.000	-15.000
0.045	-2.000	0.000	-0.007	0.000	-14.000
0.046	1.141	-0.001	0.000	0.000	-13.000
0.047	0.000	-0.001	-0.012	0.000	-12.000
0.048	-2.004	0.000	0.000	0.000	-11.000
0.049	1.141	-0.001	-0.017	0.000	-10.000
0.050	0.000	-0.001	0.000	0.000	-9.000
0.051	-2.000	0.000	-0.007	0.000	-8.000
0.052	1.141	-0.001	0.000	0.000	-7.000
0.053	0.000	-0.001	-0.012	0.000	-6.000
0.054	-2.004	0.000	0.000	0.000	-5.000
0.055	1.141	-0.001	-0.017	0.000	-4.000
0.056	0.000	-0.001	0.000	0.000	-3.000
0.057	-2.000	0.000	-0.007	0.000	-2.000
0.058	1.141	-0.001	0.000	0.000	-1.000
0.059	0.000	-0.001	-0.012	0.000	0.000
0.060	-2.004	0.000	0.000	0.000	0.000
0.061	1.141	-0.001	-0.017	0.000	0.000
0.062	0.000	-0.001	0.000	0.000	0.000
0.063	-2.000	0.000	-0.007	0.000	0.000
0.064	1.141	-0.001	0.000	0.000	0.000
0.065	0.000	-0.001	-0.012	0.000	0.000
0.066	-2.004	0.000	0.000	0.000	0.000
0.067	1.141	-0.001	-0.017	0.000	0.000
0.068	0.000	-0.001	0.000	0.000	0.000
0.069	-2.000	0.000	-0.007	0.000	0.000
0.070	1.141	-0.001	0.000	0.000	0.000
0.071	0.000	-0.001	-0.012	0.000	0.000
0.072	-2.004	0.000	0.000	0.000	0.000
0.073	1.141	-0.001	-0.017	0.000	0.000
0.074	0.000	-0.001	0.000	0.000	0.000
0.075	-2.000	0.000	-0.007	0.000	0.000
0.076	1.141	-0.001	0.000	0.000	0.000
0.077	0.000	-0.001	-0.012	0.000	0.000
0.078	-2.004	0.000	0.000	0.000	0.000
0.079	1.141	-0.001	-0.017	0.000	0.000
0.080	0.000	-0.001	0.000	0.000	0.000
0.081	-2.000	0.000	-0.007	0.000	0.000
0.082	1.141	-0.001	0.000	0.000	0.000
0.083	0.000	-0.001	-0.012	0.000	0.000
0.084	-2.004	0.000	0.000	0.000	0.000
0.085	1.141	-0.001	-0.017	0.000	0.000
0.086	0.000	-0.001	0.000	0.000	0.000
0.087	-2.000	0.000	-0.007	0.000	0.000
0.088	1.141	-0.001	0.000	0.000	0.000
0.089	0.000	-0.001	-0.012	0.000	0.000
0.090	-2.004	0.000	0.000	0.000	0.000
0.091	1.141	-0.001	-0.017	0.000	0.000
0.092	0.000	-0.001	0.000	0.000	0.000
0.093	-2.000	0.000	-0.007	0.000	0.000
0.094	1.141	-0.001	0.000	0.000	0.000
0.095	0.000	-0.001	-0.012	0.000	0.000
0.096	-2.004	0.000	0.000	0.000	0.000
0.097	1.141	-0.001	-0.017	0.000	0.000
0.098	0.000	-0.001	0.000	0.000	0.000
0.099	-2.000	0.000	-0.007	0.000	0.000
0.100	1.141	-0.001	0.000	0.000	0.000

SERIES II CASE I

FILE	Q A/C	A/C	THETA DOP	THETA	X(1) A/C	Z(1) A/C
MAXIMUM	2.000	-2.231	-1.062	-0.067	-1.635	-45.703
MINIMUM	14.571	3.735	0.163	0.038	33.652	-33.500
2.000	2.000	0.000	0.000	0.000	0.000	-33.500
2.000	-0.341	-1.001	0.000	-0.000	-0.042	-34.453
1.000	-0.020	-1.000	0.000	0.000	-0.493	-35.588
1.000	0.000	-1.000	-0.000	-0.000	-0.508	-36.746
2.000	-1.001	-1.000	0.000	-0.000	-0.488	-37.904
3.000	-0.000	-1.000	0.000	0.000	-1.000	-39.000
3.000	2.000	-1.000	-0.000	-0.000	-0.000	-39.913
4.000	-0.000	-1.000	0.000	-0.000	-0.100	-40.749
4.000	-0.000	-1.000	0.000	0.000	-0.000	-41.538
5.000	2.000	-0.000	-0.000	0.000	-0.000	-42.213
6.000	2.000	-0.000	-0.000	-0.000	1.000	-42.679
6.000	2.000	-0.000	0.000	-0.000	1.000	-43.000
7.000	2.000	-0.000	0.000	0.000	1.000	-43.474
7.000	4.000	-0.000	-0.000	0.000	3.000	-43.649
8.000	3.000	-0.000	-0.000	-0.000	5.000	-43.696
9.000	0.000	0.000	0.000	-0.000	7.000	-43.696
10.000	3.000	0.000	-0.000	0.000	7.000	-43.634
11.000	6.000	0.000	-0.000	-0.000	11.000	-43.337
12.000	3.000	0.000	0.000	-0.000	13.000	-42.969
13.000	1.000	0.000	0.000	0.000	15.000	-42.007
14.000	6.000	1.000	-0.000	0.000	17.000	-42.000
15.000	3.000	0.000	-0.000	-0.000	21.000	-41.284
16.000	4.000	1.000	0.000	-0.000	25.000	-40.405
17.000	5.000	1.000	0.000	0.000	27.000	-39.569
18.000	6.000	1.000	-0.000	0.000	31.000	-38.635
19.000	9.000	1.000	-0.000	-0.000	33.000	-38.000

SERIES II CASE 2





TIME	U A/C	W A/C	THETA DOT	THETA	X(U) A/C	Z(U) A/C
MINIMUM	2.222	-8.246	-2.167	-2.331	-32.664	-42.375
MAXIMUM	14.722	1.528	2.139	2.349	2.222	-33.522
2.222	2.222	2.222	2.222	2.222	2.222	-33.522
2.322	-1.179	-1.711	2.264	2.212	-2.257	-34.422
1.222	2.322	-1.629	-2.252	2.235	-2.212	-35.422
1.222	2.227	-1.227	-2.245	-2.222	-2.248	-36.358
2.422	-2.222	-1.324	2.127	2.229	-1.321	-37.213
3.222	2.112	-1.113	-2.257	2.243	-2.216	-37.992
3.222	-2.127	-2.222	-2.222	-2.222	-2.242	-38.613
4.222	-2.222	-2.222	2.122	-2.222	-2.127	-39.158
4.222	-1.127	-2.222	-2.218	2.218	-4.278	-39.636
5.222	2.222	-2.222	-2.222	-2.222	-4.221	-39.952
6.222	-2.222	-2.222	2.222	-2.222	-5.222	-42.197
6.222	-2.222	-2.222	2.222	2.222	-7.222	-42.356
7.222	2.222	2.222	-2.222	-2.222	-2.222	-42.359
7.222	-2.222	2.222	2.222	-2.222	-2.222	-42.284
8.222	-2.222	2.222	2.222	2.222	-11.222	-42.141
9.222	2.222	2.222	-2.222	-2.222	-12.222	-39.851
12.222	-2.222	2.222	2.222	-2.222	-13.222	-39.462
12.222	-2.222	2.222	2.222	2.222	-15.222	-39.997
12.222	-2.222	2.222	-2.222	-2.222	-17.222	-38.413
12.222	-2.222	2.222	2.222	2.222	-17.222	-37.712
12.222	-2.222	2.222	-2.222	-2.222	-22.222	-36.931
12.222	-2.222	2.222	2.222	2.222	-22.222	-36.262
12.222	-2.222	2.222	-2.222	-2.222	-23.222	-35.245
12.222	-2.222	2.222	2.222	2.222	-25.222	-33.952
12.222	-2.222	2.222	-2.222	-2.222	-29.222	-33.721
12.222	-2.222	2.222	2.222	2.222	-32.222	-34.491

SERIES II CASE 4



TIME	U A/C	I A/C	THETA DOT	THETA	X(I) A/C	Z(I) A/C
MINIMUM	0.000	-1.941	-0.144	-0.033	-21.944	-42.291
MAXIMUM	14.700	1.443	0.139	0.046	0.000	-33.500
0.000	0.000	0.000	0.000	0.000	0.000	-33.500
0.000	-1.340	-1.057	0.069	0.010	-0.266	-34.4
1.000	0.270	-1.712	-0.047	0.032	-0.940	-35.481
1.000	-0.124	-1.651	-0.035	-0.020	-0.418	-36.494
2.000	-0.000	-1.502	0.125	0.013	-1.450	-37.444
3.000	0.908	-1.290	-0.087	0.037	-2.459	-39.330
4.000	-0.040	-1.206	-0.048	-0.030	-1.957	-39.094
5.000	-0.077	-1.100	0.130	-0.033	-3.084	-39.791
6.000	-0.020	-0.933	-0.040	0.014	-4.614	-40.423
7.000	0.017	-0.782	-0.031	-0.023	-4.231	-40.917
8.000	-0.124	-0.600	0.104	-0.010	-5.151	-41.363
9.000	-1.024	-0.350	0.000	0.046	-7.063	-41.739
10.000	0.402	-0.104	-0.114	-0.013	-6.960	-41.976
11.000	-2.000	-0.200	0.072	-0.020	-7.679	-42.148
12.000	-2.040	-0.200	0.087	0.042	-9.806	-42.277
13.000	0.010	0.000	-0.142	0.031	-10.113	-42.263
14.000	-2.410	0.271	0.039	-0.031	-10.637	-42.176
15.000	-0.723	0.103	0.105	0.054	-12.840	-42.040
16.000	0.000	0.000	-0.141	0.016	-13.027	-41.780
17.000	-2.000	0.000	0.000	-0.033	-14.035	-41.431
18.000	-4.420	0.076	0.104	0.021	-16.179	-41.024
19.000	-2.000	0.000	-0.113	0.030	-17.617	-40.531
20.000	-1.000	1.122	-0.028	-0.032	-17.868	-39.919
21.000	-4.000	1.115	0.108	0.005	-19.867	-39.247
22.000	-1.000	1.049	-0.070	0.040	-21.866	-38.515
23.000	0.000	1.443	-0.144	0.034	-21.944	-38.082

SERIES II CASE 5

	TIME	U A/C	A/C	THETA DOT
MINIMUM	3.333	-2.358	-1.818	-3.392
MAXIMUM	14.724	1.323	1.923	3.387
1	3.333	3.333	3.333	3.333
3	3.633	-2.363	-1.739	3.339
5	1.223	3.736	-1.818	-3.355
7	1.823	-1.633	-1.777	3.366
9	2.423	3.634	-1.537	-3.371
11	3.023	-2.359	-1.432	3.397
13	3.623	1.232	-1.189	-3.389
15	4.223	-1.932	-1.351	3.384
17	4.823	1.297	-3.813	-3.391
19	5.423	-1.742	-3.596	3.371
21	6.023	1.285	-3.443	-3.383
23	6.623	-1.333	-3.343	3.352
25	7.223	3.362	-3.122	-3.357
27	7.823	-3.939	-3.335	3.331
29	8.423	3.122	3.175	-3.328
31	9.023	-3.534	3.327	3.311
33	9.623	-3.454	3.459	3.332
35	12.199	-3.231	3.659	-3.313
37	13.794	-1.327	3.744	3.333
39	11.394	3.132	3.993	-3.331
41	11.997	-1.564	1.342	3.361
43	12.597	3.537	1.322	-3.352
45	13.197	-1.309	1.397	3.383
47	13.796	3.936	1.656	-3.373
49	14.435	-2.323	1.763	3.387
51	14.724	-3.113	1.923	-3.315

SERIES II CASE 6



	TIME	U A/C	W A/C	THETA DOT
MINIMUM	0.000	-1.991	-1.898	-0.092
MAXIMUM	0.000	1.776	0.784	0.087
	0.000	0.000	0.000	0.000
	0.000	-0.812	-1.823	0.034
	1.200	0.732	-1.893	-0.052
	1.400	-1.576	-1.897	0.065
	2.400	0.947	-1.882	-0.079
	3.000	-1.391	-1.612	0.087
	4.000	1.344	-1.422	-0.089
	4.200	-1.849	-1.290	0.093
	4.800	1.464	-1.079	-0.091
	5.400	-1.036	-0.984	0.070
	6.000	1.316	-0.771	-0.080
	6.600	-1.117	-0.698	0.062
	7.200	0.966	-0.499	-0.057
	7.800	-0.664	-0.421	0.032
	8.400	0.522	-0.268	-0.029
	8.999	-0.211	-0.147	0.011
	9.599	0.001	-0.028	0.001
	10.199	0.241	0.126	-0.010
	10.797	-0.496	0.201	0.031
	11.396	0.693	0.401	-0.030
	11.995	-0.907	0.441	0.060
	12.594	1.148	0.677	-0.051
	12.996	-0.942	0.784	0.042

SERIES II CASE 10

TIME	U A/C	V A/C	THETA DOT
14-0146	-2.109	-1.480	-0.092
14-0147	1.487	1.214	0.087
14-0148	0.000	0.000	0.000
14-0149	-0.946	-1.919	0.036
14-0150	0.000	-1.936	-0.053
14-0151	-1.004	-1.931	0.004
14-0152	0.000	-1.000	-0.070
14-0153	-2.109	-1.000	0.047
14-0154	1.100	-1.000	-0.039
14-0155	-2.019	-1.272	0.084
14-0156	1.004	-1.000	-0.052
14-0157	-1.001	-0.952	0.072
14-0158	1.104	-0.701	-0.092
14-0159	-1.007	-0.072	0.004
14-0200	0.000	-0.429	-0.060
14-0201	-0.012	-0.304	0.034
14-0202	0.000	-0.033	-0.033
14-0203	-0.016	-0.119	0.014
14-0204	-0.000	-0.000	-0.000
14-0205	-0.001	0.100	-0.007
14-0206	-0.007	0.225	0.028
14-0207	0.004	0.429	-0.023
14-0208	-1.004	0.463	0.007
14-0209	0.000	0.000	-0.049
14-0210	-1.000	0.000	0.000
14-0211	1.004	0.000	-0.000
14-0212	-1.000	1.000	0.000
14-0213	0.000	0.000	0.000
14-0214	0.000	0.000	0.000

SERIES 11 CASE 11

Wavelength	U A/C	V A/C	THETA DUT	THETA	X(U) A/C	Z(U) A/C
11.722	-1.311	-1.923	-2.299	-2.319	-2.522	-43.212
11.724	2.755	1.431	2.387	2.317	8.221	-33.295
11.725	2.722	2.222	2.322	2.322	2.322	-33.522
11.726	2.722	-1.987	-2.322	2.322	2.322	-33.122
11.727	2.722	-2.239	2.335	-2.322	2.322	-33.749
11.728	2.722	-2.272	2.323	-2.321	-2.341	-33.933
11.729	2.722	2.276	-2.322	2.314	-2.322	-33.792
11.730	2.722	-1.779	2.366	2.321	-2.237	-34.579
11.731	2.722	-1.620	-2.395	2.322	-2.271	-35.617
11.732	2.722	-1.242	2.372	-2.312	2.343	-36.981
11.733	2.722	-1.339	-2.382	2.327	2.249	-37.523
11.734	2.722	-1.349	2.392	-2.314	2.998	-38.631
11.735	2.722	-1.149	-2.358	2.311	2.725	-39.119
11.736	2.722	-1.119	2.332	-2.317	1.522	-39.227
11.737	2.722	-2.982	-2.337	2.314	1.313	-40.406
11.738	2.722	-2.923	2.312	-2.318	2.322	-41.222
11.739	2.722	-2.939	-2.322	2.315	2.279	-41.548
11.740	2.722	-2.989	-2.329	-2.319	3.151	-41.969
11.741	2.722	-2.915	2.329	2.314	3.223	-42.352
11.742	2.722	-2.472	-2.328	-2.317	4.299	-42.652
11.743	2.722	-2.426	2.358	2.312	4.296	-42.924
11.744	2.722	-2.252	-2.349	-2.315	5.145	-43.265
11.745	2.722	-2.216	2.373	2.327	5.323	-43.175
11.746	2.722	-2.226	-2.363	-2.311	6.289	-43.227
11.747	2.722	2.231	2.386	2.323	6.634	-43.181
11.748	2.722	2.229	-2.390	-2.327	7.557	-43.294
11.749	2.722	2.222	2.396	-2.323	8.137	-42.919
11.750	2.722	2.379	2.326	2.315	8.221	-42.833

SERIES II CASE 14

	TIME	U A/C	V A/C	THETA DOT	THETA	X(I) A/C	Z(I) A/C
MINIMUM	3.333	-2.246	-1.991	-3.332	-3.318	-2.444	-46.725
MAXIMUM	14.835	1.032	3.532	3.387	3.316	3.333	-33.523
	3.333	2.333	3.333	3.333	3.333	3.333	-33.533
	3.333	-3.333	-1.953	3.332	3.334	-3.163	-34.439
	1.233	3.331	-1.959	-3.331	3.331	-3.339	-35.584
	1.833	-1.337	-1.991	3.364	3.311	-3.728	-36.759
	2.433	3.333	-1.913	-3.372	-3.313	-3.636	-37.971
	3.033	-2.246	-1.774	3.337	3.331	-1.381	-38.926
	3.633	1.231	-1.579	-3.339	-3.335	-1.199	-39.915
	4.233	-1.334	-1.133	3.384	-3.334	-1.423	-42.833
	4.833	1.143	-1.317	-3.332	3.331	-1.639	-41.693
	5.433	-1.377	-1.213	3.372	-3.313	-1.557	-42.472
	6.033	1.333	-1.263	-3.331	3.337	-2.383	-43.232
	6.633	-1.143	-1.217	3.354	-3.314	-1.531	-43.846
	7.233	1.249	-2.313	-3.363	3.311	-2.335	-44.446
	7.833	-1.631	-3.737	3.334	-3.317	1.855	-44.944
	8.433	3.324	-2.655	-3.333	3.314	-2.414	-45.413
	9.033	-2.212	-3.593	3.314	-3.313	-1.821	-45.783
	9.633	3.163	-3.491	-3.333	3.316	-2.436	-46.121
	10.233	3.237	-3.364	3.336	-3.318	-1.699	-46.355
	10.833	-3.336	-3.327	-3.327	3.315	-2.213	-46.563
	11.433	3.723	-3.147	3.327	-3.317	-1.487	-46.561
	12.033	-3.763	-3.123	3.356	3.312	-1.892	-46.728
	12.633	1.133	3.372	-3.347	-3.315	-1.188	-46.721
	13.233	-1.362	3.393	3.377	3.338	-1.431	-46.625
	13.833	1.675	3.293	-3.367	-3.311	-3.831	-46.473
	14.433	-1.152	3.343	3.387	3.332	-3.789	-46.252
	15.033	3.133	3.532	-3.321	3.315	-3.879	-46.135

SERIES II CASE 15

	TIME	U A/C	V A/C	THETA DOT
MINIMUM	0.000	-1.987	-1.973	-0.092
MAXIMUM	14.704	1.751	0.530	0.087
	0.000	0.000	0.000	0.000
	0.000	-0.810	-1.842	0.034
	1.000	0.705	-1.946	-0.052
	1.800	-1.549	-1.973	0.064
	2.400	0.902	-1.793	-0.070
	3.000	-1.987	-1.747	0.087
	3.600	1.309	-1.562	-0.088
	4.200	-1.892	-1.482	0.094
	4.800	1.462	-1.300	-0.092
	5.400	-1.003	-1.229	0.073
	6.000	1.350	-1.045	-0.082
	6.600	-1.211	-0.907	0.055
	7.200	1.029	-0.822	-0.062
	7.800	-0.774	-0.776	0.035
	8.400	0.579	-0.633	-0.035
	9.000	-0.836	-0.558	0.015
	9.600	0.285	-0.459	-0.005
	10.200	0.101	-0.341	-0.005
	10.800	-0.411	-0.280	0.024
	11.400	0.540	-0.124	-0.025
	11.997	-0.900	-0.109	0.054
	12.597	0.980	0.094	-0.045
	13.195	-1.247	0.107	0.075
	13.793	1.368	0.317	-0.063
	14.391	-1.566	0.355	0.085
	14.989	0.753	0.530	-0.028

SERIES IJ CASE 16



# SERIES III

CASE	THRUST	SWH	$\theta_t$	$\delta$	$\mu$	$T_B$	
1	33650	30	-.022	94°	.68	0	
2	33650	10	-.0205	4°	.15	5	
3	33600	30	-.013	4°	.68	5	(1)
4	33650	30	-.019	4°	.68	5	
5	33750	30	-.017	4°	.68	5	
6	33700	30	-.021	94	.68	5	
7	33800	30	-.0205	64	.68	3	
8	33800	30	-.0205	64	.68	5	(1)
9	33800	30	-.014	64	.68	0	
10	338500	30	-.0192	49	.68	5	
11	33850	30	-.0232	94	.3	5	(2)

1. 20 KT. HEADWIND

2. DROP HEIGHT = 16" FOR ALL OTHER CASES  
DROP HEIGHT = 7.5"

TIME	U A/C	W A/C	THETA DOT	THETA	X(1) A/C	Z(1) A/C
14.000	-0.004	-4.902	-0.112	-0.016	-1.189	-76.615
14.000	0.101	2.253	0.231	0.157	30.331	-33.300
0.000	0.000	-3.153	-0.001	0.071	0.000	-33.500
0.000	-0.004	-1.646	0.002	0.157	0.000	-33.300
1.000	-0.004	-3.713	-0.029	0.133	-0.352	-35.777
1.000	-0.004	-2.935	0.015	0.136	-0.597	-37.605
2.000	-0.004	-3.431	-0.041	0.123	-0.873	-39.627
3.000	0.002	-2.731	0.007	0.108	-1.125	-41.793
4.000	0.000	-3.136	-0.047	0.116	-1.129	-43.220
5.000	2.004	-3.763	-0.079	0.075	-0.361	-45.451
6.000	0.104	-4.004	-0.102	0.016	2.217	-48.070
7.000	1.701	-4.455	0.065	0.008	4.183	-50.803
8.000	4.001	-4.227	-0.095	0.002	6.020	-53.422
9.000	1.504	-4.053	0.077	-0.005	7.851	-55.905
10.000	1.100	-3.727	-0.086	0.008	9.318	-58.273
11.000	1.003	-3.619	0.060	-0.010	11.347	-60.497
12.000	4.124	-3.264	-0.067	0.013	12.624	-62.509
13.000	2.077	-3.190	0.033	-0.013	14.814	-64.561
14.000	3.000	-2.844	-0.039	0.017	15.907	-66.411
15.000	2.004	-2.757	0.018	-0.016	13.248	-68.035
16.000	3.016	-2.448	-0.008	0.018	19.319	-69.679
17.000	2.001	-2.320	0.004	-0.016	21.648	-71.093
18.000	2.416	-2.058	0.005	0.019	22.715	-72.416
19.000	3.240	-1.377	-0.005	-0.015	25.010	-73.566
20.000	1.007	-1.603	0.004	0.015	26.138	-74.611
21.000	3.003	-1.427	-0.006	-0.012	23.335	-75.503
22.000	1.005	-1.259	0.006	0.011	29.584	-76.276
23.000	3.707	-0.942	-0.000	0.015	30.331	-76.615

SERIES III CASE 1

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MINIMUM	TIME	WHEEL GEAR	MAIN GEAR	ZLG	NGL	NGL
	2.000	2.000	2.000	-53059.102	1.755	3.463
MAXIMUM	14.700	879.104	44270.937	0.000	12.000	12.000
2.000	0463.353	0.000	0.000	-5433.833	12.000	7.560
3.000	8312.777	38196.520	0.000	-46499.102	1.755	3.463
1.000	9448.421	11136.449	0.000	-17634.871	7.205	5.743
1.000	5534.751	11353.262	0.000	-10987.992	7.368	7.015
2.400	4570.261	7605.782	0.000	-12275.243	9.721	9.237
3.000	5770.434	0.000	0.000	-3979.434	12.000	11.268
4.000	0.000	7227.313	0.000	-7227.313	11.799	12.000
4.000	0.000	0.000	0.000	0.000	12.000	12.000
5.000	0.000	0.000	0.000	0.000	12.000	12.000
6.000	0.000	0.000	0.000	0.000	12.000	12.000
6.000	0.000	0.000	0.000	0.000	12.000	12.000
7.000	0.000	0.000	0.000	0.000	12.000	12.000
7.000	0.000	0.000	0.000	0.000	12.000	12.000
8.000	0.000	0.000	0.000	0.000	12.000	12.000
9.000	0.000	0.000	0.000	0.000	12.000	12.000
9.000	0.000	0.000	0.000	0.000	12.000	12.000
10.000	0.000	0.000	0.000	0.000	12.000	12.000
10.000	0.000	0.000	0.000	0.000	12.000	12.000
11.000	0.000	0.000	0.000	0.000	12.000	12.000
11.000	0.000	0.000	0.000	0.000	12.000	12.000
12.000	0.000	0.000	0.000	0.000	12.000	12.000
12.000	0.000	0.000	0.000	0.000	12.000	12.000
13.000	0.000	0.000	0.000	0.000	12.000	12.000
13.000	0.000	0.000	0.000	0.000	12.000	12.000
14.000	0.000	0.000	0.000	0.000	12.000	12.000
14.000	0.000	0.000	0.000	0.000	12.000	12.000

SERIES III CASE 1 (Cont'd)

MINIMUM	TIME	ZERO S CG	Z-PAW CP	Z-REAR CP	ZF	BIS	GAMMA
	2-000	-14.296	-44.785	-44.769	-35649.633	-0.192	-1.417
MAXIMUM	14.700	0.000	-27.913	-26.000	0.000	0.294	1.154
2-000	2-000	0.000	-27.913	-26.000	0.000	0.000	-1.264
3-000	3-000	-1.000	-29.755	-27.896	-6052.793	0.284	-1.024
4-000	4-000	-2.000	-31.597	-29.751	-11931.156	0.284	-1.000
5-000	5-000	-3.000	-33.439	-31.591	-17308.535	0.284	-1.000
6-000	6-000	-4.000	-35.281	-33.435	-22112.199	0.284	-1.000
7-000	7-000	-5.000	-37.123	-35.280	-26133.023	0.284	-1.000
8-000	8-000	-6.000	-38.965	-37.123	-29227.914	0.284	-1.000
9-000	9-000	-7.000	-40.807	-38.965	-31287.344	0.284	-1.000
10-000	10-000	-8.000	-42.649	-40.807	-33390.336	-0.107	-0.729
11-000	11-000	-9.000	-44.491	-42.649	-35507.148	-0.000	-1.000
12-000	12-000	-10.000	-46.333	-44.491	-37612.086	-0.047	-0.737
13-000	13-000	-11.000	-48.175	-46.333	-39717.024	-0.121	-1.040
14-000	14-000	-12.000	-50.017	-48.175	-41822.062	0.113	-0.694
15-000	15-000	-13.000	-51.859	-50.017	-43927.100	-0.175	-1.132
16-000	16-000	-14.000	-53.701	-51.859	-46032.138	0.215	-0.669
17-000	17-000	-15.000	-55.543	-53.701	-48137.176	-0.192	-0.994
18-000	18-000	-16.000	-57.385	-55.543	-50242.214	0.273	-0.669
19-000	19-000	-17.000	-59.227	-57.385	-52347.252	-0.192	-0.841
20-000	20-000	-18.000	-61.069	-59.227	-54452.290	0.284	-0.692
21-000	21-000	-19.000	-62.911	-61.069	-56557.328	-0.192	-0.693
22-000	22-000	-20.000	-64.753	-62.911	-58662.366	0.284	-0.706
23-000	23-000	-21.000	-66.595	-64.753	-60767.404	-0.192	-0.525
24-000	24-000	-22.000	-68.437	-66.595	-62872.442	0.261	-0.743
25-000	25-000	-23.000	-70.279	-68.437	-64977.480	-0.192	-0.374
26-000	26-000	-24.000	-72.121	-70.279	-67082.518	0.158	-0.734
27-000	27-000	-25.000	-73.963	-72.121	-69187.556	0.257	-0.247

SERIES III CASE 1 (Cont'd)

TIME	U A/C	V A/C	THETA DOF	THETA	X(U) A/C	Z(U) A/C
0.000	-2.400	-2.317	-0.094	-0.017	-0.002	-33.930
0.005	0.003	1.005	0.007	0.021	0.005	-33.061
0.010	0.000	-0.232	0.001	-0.002	0.000	-33.500
0.015	0.000	0.454	0.010	0.010	0.002	-33.064
0.020	0.000	-1.215	-0.019	-0.003	0.002	-33.591
0.025	0.000	0.762	0.071	0.005	0.003	-33.055
0.030	0.000	-0.319	-0.020	0.021	0.006	-33.419
0.035	0.000	-1.714	-0.034	-0.000	0.001	-33.932
0.040	-0.000	-0.40	0.005	0.005	0.001	-34.391
0.045	-0.034	-0.200	0.019	0.008	-0.004	-34.353
0.050	-0.134	-1.22	-0.024	0.001	-0.009	-34.002
0.055	-0.135	0.310	0.005	0.010	-0.006	-34.741
0.060	-1.005	-1.243	-0.061	0.009	-0.015	-34.555
0.065	-0.101	-1.000	0.019	-0.002	-0.003	-35.299
0.070	-1.134	-1.000	0.019	0.010	-1.011	-35.977
0.075	-1.432	-1.000	0.004	-0.001	-1.002	-36.064
0.080	0.004	-1.397	-0.004	-0.016	-1.836	-37.094
0.085	-1.400	-1.222	0.024	0.011	-2.718	-37.571
0.090	-2.225	-1.106	0.063	0.005	-2.967	-37.956
0.095	0.705	-0.046	-0.008	-0.017	-2.919	-38.289
0.100	-0.006	-0.010	0.003	0.006	-3.001	-38.557
0.105	-2.400	-0.062	0.004	0.010	-4.142	-38.709
0.110	0.457	-0.026	-0.071	-0.017	-3.980	-38.976
0.115	-0.027	-0.219	-0.019	-0.000	-4.621	-38.936
0.120	-2.400	-0.106	0.007	0.000	-5.236	-38.945
0.125	-0.009	0.192	-0.044	0.015	-5.017	-38.854
0.130	-0.000	0.202	-0.041	-0.015	-5.493	-38.726
0.135	-2.209	0.443	0.078	-0.006	-5.962	-38.620
0.140	-2.150	0.410	0.072	0.010		

SERIES III CASE 2

TIME	Y A/C	X A/C	THETA DOT	THETA	X(I) A/C	Z(I) A/C
MINIMUM	2.000	0.000	-0.014	-0.162	-0.151	-91.096
HALF MAX	14.000	22.170	0.258	0.101	147.944	-33.500
	2.000	0.000	0.012	-0.006	0.000	-33.500
	3.000	0.000	-0.006	0.007	-0.010	-34.963
	4.000	0.000	0.009	0.019	-0.019	-36.956
	5.000	0.000	-0.025	0.044	-0.079	-39.471
	6.000	0.000	-0.012	0.029	-0.101	-42.002
	7.000	0.000	0.003	0.000	0.075	-44.004
	8.000	0.000	-0.040	0.062	1.719	-46.011
	9.000	0.000	-0.038	0.033	4.204	-48.029
	10.000	0.000	-0.105	-0.013	9.169	-50.048
	11.000	0.000	-0.038	-0.061	15.043	-52.067
	12.000	0.000	0.146	-0.026	21.145	-54.085
	13.000	0.000	0.090	0.068	25.600	-56.097
	14.000	0.000	-0.179	0.039	32.013	-58.109
	15.000	0.000	-0.183	-0.006	40.229	-60.115
	16.000	0.000	0.034	-0.148	47.801	-62.121
	17.000	0.000	0.191	-0.088	54.769	-64.134
	18.000	0.000	0.232	0.054	60.990	-66.143
	19.000	0.000	-0.039	0.092	68.786	-68.157
	20.000	0.000	-0.276	-0.025	78.814	-70.167
	21.000	0.000	-0.109	-0.144	89.208	-72.179
	22.000	0.000	0.078	-0.152	99.966	-74.196
	23.000	0.000	0.256	-0.048	108.136	-76.212
	24.000	0.000	0.121	0.046	117.166	-78.224
	25.000	0.000	-0.150	0.075	128.259	-80.238
	26.000	0.000	-0.245	-0.070	141.360	-82.251
	27.000	0.000	-0.152	-0.129	147.844	-84.265

SERIES III CASE 3

MIXTURE	TIME	U A/C	W A/C	THETA DOT	THETA	X(I) A/C	Z(I) A/C
	0.000	0.000	-4.642	-0.093	-0.019	-0.150	-77.885
	14.700	10.990	-1.013	0.097	0.071	99.454	-33.500
	0.000	0.000	-3.153	0.012	-0.006	0.000	-33.500
	0.000	0.000	-4.483	-0.001	0.007	-0.009	-34.945
	1.000	0.000	-2.121	0.006	0.017	-0.017	-36.986
	1.000	0.000	-3.022	-0.004	0.041	-0.000	-38.527
	2.000	0.007	-3.630	-0.012	0.033	-0.150	-40.792
	3.000	1.492	-1.001	0.080	0.000	0.012	-42.219
	3.000	2.452	-3.005	-0.011	0.004	1.409	-43.996
	4.000	6.271	-2.278	-0.036	0.048	4.010	-46.023
	4.000	12.047	-4.002	-0.001	0.015	8.870	-48.420
	5.000	7.707	-4.496	0.006	0.001	14.432	-51.167
	6.000	10.040	-4.216	-0.003	0.005	19.630	-53.821
	6.000	9.010	-4.204	0.002	-0.012	25.215	-56.344
	7.000	10.239	-3.929	-0.070	0.009	30.129	-58.764
	7.000	9.103	-3.810	0.043	-0.016	35.945	-61.023
	8.000	9.942	-3.290	-0.044	0.013	40.798	-63.189
	9.000	8.410	-3.407	0.022	-0.018	46.801	-65.189
	9.000	9.440	-2.833	-0.015	0.015	51.650	-67.000
	10.000	9.113	-2.990	0.001	-0.019	57.780	-68.834
	10.000	8.373	-2.500	0.016	0.016	62.680	-70.481
	11.000	9.000	-2.007	-0.020	-0.018	68.870	-71.962
	12.000	8.491	-2.141	0.047	0.013	73.895	-73.348
	12.000	10.145	-2.107	-0.042	-0.016	80.063	-74.568
	13.000	8.117	-1.709	0.072	0.009	85.264	-75.685
	13.000	10.001	-1.009	-0.003	-0.012	91.361	-76.605
	14.000	7.000	-1.043	0.085	0.003	96.794	-77.500
	14.000	10.000	-1.013	-0.020	0.010	99.454	-77.885

SERIES III CASE 4

MINIMUM	TIME	P/F	IF	HOSE GEAR	MAIN GEAR	7LG
	0-000	-33633-049	-6421-178	0-000	0-000	-35774-117
MAXIMUM	14-700	0-000	9424-048	9535-971	27134-145	0-000
0-000	0-000	0-000	0-000	5433-965	12424-002	-17937-967
0-000	-0122-213	0-000	1233-934	8441-078	21721-105	-33162-795
1-700	-1150-145	0-000	3470-927	6197-313	13116-193	-19313-512
1-400	-1703-012	0-000	5552-108	5675-994	13102-950	-18779-940
2-400	-2011-090	0-000	6154-229	4035-859	6148-573	-13835-437
3-000	-2013-062	0-000	7527-958	4017-472	9025-454	-12022-926
3-000	-2322-049	0-000	8531-003	5957-770	0-000	-3359-220
4-200	-3127-137	0-000	9132-416	4072-564	0-000	-4072-364
4-000	-3320-039	0-000	4335-530	3353-756	0-000	-3853-756
5-400	-3441-414	0-000	-3779-959	0-000	0-000	0-000
6-000	-3554-533	0-000	2534-000	0-000	0-000	0-000
6-000	-3547-492	0-000	-5654-569	0-000	0-000	0-000
7-200	-32364-023	0-000	6759-991	0-000	0-000	0-000
7-000	-3323-727	0-000	-6410-563	0-000	0-000	0-000
8-100	-32454-010	0-000	8310-277	0-000	0-000	0-000
9-000	-33231-004	0-000	-6421-178	0-000	0-000	0-000
9-000	-3232-202	0-000	9424-048	0-000	0-000	0-000
10-200	-33231-004	0-000	-6421-178	0-000	0-000	0-000
10-000	-3232-202	0-000	9424-049	0-000	0-000	0-000
11-400	-33231-004	0-000	-6421-178	0-000	0-000	0-000
12-000	-3232-202	0-000	9335-965	0-000	0-000	0-000
12-000	-33231-004	0-000	-6421-178	0-000	0-000	0-000
13-200	-33234-009	0-000	6239-828	0-000	0-000	0-000
13-000	-33231-004	0-000	-6121-178	0-000	0-000	0-000
14-400	-33231-004	0-000	2320-324	0-000	0-000	0-000
14-700	-3231A-572	0-000	9371-541	0-000	0-000	0-000

SERIES III CASE 4 (Cont'd)



MINIMUM	TIME	Z-10 S CG	Z-FWD CP	2-REAR CP	HGL	HGL	NGL
	2-000	-13.296	-49.956	-49.550	2.951		3.414
	14-000	2.000	-26.389	-26.539	12.000		12.000
	2-000	2.000	-26.339	-26.539	6.616		7.560
	3-000	-1.999	-29.424	-29.376	2.951		3.414
	1-000	-3.757	-32.438	-32.193	6.504		6.153
	1-000	-5.595	-32.429	-31.969	5.725		6.721
	2-000	-7.354	-34.316	-33.636	11.488		8.991
	3-000	-9.244	-35.154	-35.341	11.542		11.051
	3-000	-12.113	-37.919	-36.931	12.000		11.851
	4-000	-12.119	-39.042	-39.487	12.000		11.661
	4-000	-13.463	-41.359	-42.034	12.000		11.825
	5-000	-14.876	-43.229	-41.567	12.000		12.000
	6-000	-15.725	-44.516	-42.950	12.000		12.000
	6-000	-16.027	-45.898	-44.226	12.000		12.000
	7-000	-17.312	-47.269	-45.339	12.000		12.000
	7-000	-17.932	-48.129	-46.281	12.000		12.000
	8-000	-19.151	-49.978	-47.200	12.000		12.000
9-000		-19.296	-49.803	-47.596			12.000
	9-000	-18.236	-49.809	-47.892			
	10-000	-17.942	-49.956	-48.250			
	12-000	-17.535	-49.758	-47.894			
	11-000	-16.921	-49.394	-47.561			
	12-000	-16.240	-48.661	-46.929			
	12-000	-15.122	-47.742	-46.280			
	13-000	-13.333	-46.496	-44.941			
	13-000	-12.656	-45.239	-43.529			
	14-000	-11.225	-43.293	-42.032			
	14-000	-12.433	-42.329	-41.112			

SERIES III CASE 4 (Cont'd)



MIXTURE	FILE	W A/C	W A/C	THETA OUT	THETA	K(I) A/C	Z(I) A/C
	7-222	-2-219	-6-247	-3-275	-3-154	-1-254	-94-538
MIXTURE	11-511	13-522	3-725	3-334	3-183	81-119	-33-244
2-222	2-222	2-222	-3-153	-3-331	3-371	3-333	-33-533
3-222	3-222	3-222	-2-223	3-365	3-163	3-317	-33-244
1-222	1-222	1-222	-3-493	-3-323	3-133	-3-373	-33-829
1-222	1-222	1-222	-2-136	3-322	3-143	-3-611	-37-524
2-422	2-422	2-422	-4-274	-3-353	3-129	-3-912	-39-639
3-422	3-422	3-422	-2-277	3-353	3-137	-1-173	-41-823
3-422	3-422	3-422	-3-333	-3-326	3-113	-1-194	-43-237
4-422	4-422	4-422	-4-254	-3-363	3-372	-3-544	-45-531
4-422	4-422	4-422	-4-941	-3-116	3-339	2-342	-48-233
5-422	5-422	5-422	-3-435	-3-377	-3-373	5-939	-51-199
6-422	6-422	6-422	-3-231	3-139	-3-359	8-739	-54-233
6-422	6-422	6-422	-4-918	3-183	3-348	13-163	-57-233
7-222	7-222	7-222	-1-254	-3-372	3-384	12-541	-53-147
7-222	7-222	7-222	-4-916	-3-256	-3-332	17-173	-62-946
8-422	8-422	8-422	-3-633	-3-381	-3-134	21-946	-55-732
9-422	9-422	9-422	-3-511	3-133	-3-125	25-354	-68-587
9-422	9-422	9-422	-4-932	3-263	-3-337	29-112	-71-449
12-222	12-222	12-222	-3-331	3-361	3-133	32-481	-74-322
12-422	12-422	12-422	-3-483	-3-229	3-356	37-973	-77-373
12-422	12-422	12-422	-3-479	-3-199	-3-333	45-326	-79-743
12-422	12-422	12-422	-3-247	-3-312	-3-154	151-356	-83-443
12-422	12-422	12-422	-3-362	3-175	-3-133	57-133	-85-234
13-422	13-422	13-422	-4-344	3-246	3-743	62-376	-88-212
13-422	13-422	13-422	-3-153	-3-321	3-139	63-133	-93-762
14-422	14-422	14-422	-4-371	-3-267	3-315	70-463	-93-391
14-422	14-422	14-422	-3-322	-3-241	-3-356	81-119	-94-693

SERIES III CASE 6

TIME	U A/C	V A/C	THETA OUT	THETA	X(1) A/C	Z(1) A/C
0.000	0.000	-4.743	-0.003	-0.016	-0.042	-00.993
0.001	0.001	-1.445	0.004	0.044	30.041	-33.500
0.002	0.002	-3.153	0.012	-0.006	0.000	-33.500
0.003	0.003	-1.445	0.007	0.003	0.001	-33.503
0.004	0.004	-3.350	-0.009	0.012	-0.004	-31.444
0.005	0.005	-4.473	-0.025	0.008	-0.012	-30.000
0.006	0.006	-3.341	0.020	0.003	-0.014	-30.181
0.007	0.007	-3.139	0.005	0.011	-0.019	-37.290
0.008	0.008	-3.191	0.018	0.017	-0.029	-37.026
0.009	0.009	-3.410	-0.001	0.019	-0.042	-34.840
0.010	0.010	-3.211	0.004	0.022	0.000	-39.600
0.011	0.011	-2.934	0.002	0.031	0.011	-40.341
0.012	0.012	-3.974	0.002	0.036	1.329	-40.060
0.013	0.013	-2.974	0.003	0.042	2.000	-41.000
0.014	0.014	-2.933	-0.003	0.044	4.005	-42.404
0.015	0.015	-3.403	-0.006	0.040	6.002	-43.206
0.016	0.016	-4.101	-0.041	0.030	8.432	-44.261
0.017	0.017	-4.021	-0.004	0.015	11.196	-45.006
0.018	0.018	-4.009	-0.007	-0.002	14.109	-46.515
0.019	0.019	-4.400	0.018	-0.007	16.333	-47.000
0.020	0.020	-4.725	0.003	0.000	19.201	-48.010
0.021	0.021	-4.002	0.016	0.019	21.043	-49.007
0.022	0.022	-4.400	-0.003	0.000	24.000	-51.000
0.023	0.023	-4.737	-0.001	-0.011	27.275	-52.101
0.024	0.024	-4.704	0.014	-0.016	29.000	-53.000
0.025	0.025	-4.000	0.001	-0.004	32.040	-54.000
0.026	0.026	-4.004	0.000	0.014	34.000	-55.000
0.027	0.027	-1.200	-0.004	0.017	36.041	-56.000

SERIES III CASE 7



TIME	U A/C	V A/C	THETA DOT	THETA	X(U) A/C	Z(U) A/C
11-000	33.000	-3.467	-0.108	-0.016	0.000	-131.657
11-005	40.010	-1.426	0.037	0.070	605.030	-53.500
11-010	44.710	-3.103	0.012	-0.006	0.000	-33.500
11-015	44.810	-4.006	-0.035	0.011	20.270	-34.953
11-020	44.910	-2.001	0.057	0.010	42.004	-37.342
11-025	44.910	-3.244	0.017	0.042	60.704	-39.593
11-030	44.910	-3.434	-0.014	0.041	80.925	-42.635
11-035	44.910	-2.000	0.019	0.060	101.125	-45.222
11-040	44.910	-2.001	0.009	0.065	121.609	-48.394
11-045	44.910	-2.010	0.011	0.070	143.993	-51.553
11-050	44.910	-4.014	-0.034	0.068	167.000	-55.240
11-055	44.910	-7.220	-0.104	0.014	194.362	-59.090
11-060	44.910	-7.910	0.063	-0.003	221.394	-64.399
11-065	44.910	-7.410	-0.040	0.008	247.370	-69.121
11-070	44.910	-3.270	0.059	-0.010	273.973	-73.119
11-075	44.910	-7.200	-0.067	0.012	299.837	-78.529
11-080	44.910	-8.301	0.043	-0.013	326.611	-83.200
11-085	44.910	-7.002	-0.046	0.015	352.437	-87.946
11-090	44.910	-8.410	0.026	-0.015	379.374	-92.535
11-095	44.910	-5.000	-0.023	0.017	405.173	-97.204
11-100	44.910	-0.101	0.001	-0.016	432.206	-101.922
11-105	44.910	-5.004	0.001	0.017	459.122	-106.463
11-110	44.910	-8.002	-0.003	-0.016	485.273	-111.056
11-115	44.910	-6.004	0.026	0.017	511.202	-115.673
11-120	44.910	-8.101	-0.025	-0.015	538.144	-120.243
11-125	44.910	-7.000	0.049	0.015	564.519	-124.837
11-130	44.910	-3.204	-0.042	-0.014	591.752	-129.387
11-135	44.910	-8.000	0.054	-0.012	625.030	-131.657

SERIES III CASE 8

MINIMUM	TIME	NOSE GEAR	MAIN GEAR
MINIMUM	14.696	9137.838	29173.963
	0.000	5430.955	12424.002
	0.000	7474.780	23595.048
	1.200	6531.976	13637.549
	1.900	5725.016	12706.340
	2.400	4351.481	7334.003
	3.000	7407.095	10726.026
	3.600	0.000	0.000
	4.200	571.623	0.000
	4.800	4140.972	0.000
	5.400	0.000	0.000
	6.000	0.000	0.000
	6.600	0.000	0.000
	7.200	0.000	0.000
	7.800	0.000	0.000
	8.400	0.000	0.000
	9.000	0.000	0.000
	9.600	0.000	0.000
	10.200	0.000	0.000
	10.800	0.000	0.000
	11.400	0.000	0.000
	12.000	0.000	0.000
	12.600	0.000	0.000
	13.200	0.000	0.000
	13.800	0.000	0.000
	14.400	0.000	0.000
	15.000	0.000	0.000

SERIES IIT CASE 8 (Cont'd)

TIME	U A/C	W A/C	THETA DOT	THETA	X(U) A/C	Z(U) A/C
MINIMUM	3.000	3.000	-0.096	-0.019	-0.150	-87.383
MAXIMUM	14.709	10.903	0.097	0.072	97.523	-33.500
3.000	3.000	3.000	0.012	-0.006	0.000	-33.500
3.000	3.000	3.000	-0.051	0.007	-0.008	-34.041
1.000	3.000	3.000	0.005	0.016	-0.017	-36.993
1.000	3.000	3.000	-0.003	0.041	-0.009	-38.631
2.000	3.000	3.000	-0.012	0.036	-0.010	-40.782
3.000	2.047	2.047	0.004	0.009	0.001	-42.267
3.000	3.000	3.000	-0.010	0.005	1.000	-43.909
1.000	3.000	3.000	-0.006	0.047	4.014	-45.009
3.000	3.000	3.000	-0.071	0.011	9.004	-49.033
3.000	7.000	7.000	0.000	0.003	14.000	-51.333
6.000	10.000	10.000	-0.006	-0.000	19.000	-54.000
6.000	7.000	7.000	0.000	-0.009	24.000	-56.771
7.000	10.000	10.000	-0.004	0.000	29.000	-59.416
7.000	7.000	7.000	0.000	-0.013	35.000	-61.990
8.000	10.000	10.000	-0.006	0.010	40.000	-64.000
8.000	3.000	3.000	0.040	-0.016	45.000	-66.968
9.000	3.000	3.000	-0.041	0.013	50.000	-69.379
10.000	3.000	3.000	0.000	-0.018	55.000	-71.700
10.000	3.000	3.000	-0.013	0.015	61.000	-74.000
11.000	3.000	3.000	0.001	-0.019	67.000	-76.000
11.000	3.000	3.000	0.015	0.015	72.000	-79.396
12.000	3.000	3.000	-0.019	-0.019	78.000	-80.409
13.000	3.000	3.000	0.044	0.013	83.000	-82.526
13.000	10.000	10.000	-0.008	-0.017	89.000	-84.490
14.000	5.000	5.000	0.000	0.010	94.000	-86.416
14.000	10.000	10.000	-0.005	0.010	97.000	-87.383

SERIES III CASE 9



ITEM	TIME	NOSE GEAR	MAIN GEAR
PIVOT	0.000	0.000	0.000
PIVOT	14.725	9631.131	27054.328
	0.000	0483.865	12424.002
	0.000	8442.996	21974.719
	1.200	6196.542	13009.465
	1.300	5551.613	15019.528
	2.400	4893.799	6082.746
	3.000	3996.155	5025.713
	3.600	3735.062	0.000
	4.200	4028.909	0.000
	4.300	0.000	0.000
	5.400	0.000	0.000
	6.000	0.000	0.000
	6.600	0.000	0.000
	7.200	0.000	0.000
	7.800	0.000	0.000
	8.400	0.000	0.000
	9.000	0.000	0.000
	9.600	0.000	0.000
	10.200	0.000	0.000
	10.800	0.000	0.000
	11.400	0.000	0.000
	11.997	0.000	0.000
	12.596	0.000	0.000
	13.195	0.000	0.000
	13.796	0.000	0.000
	14.395	0.000	0.000
	14.725	0.000	0.000

SERIES III CASE 9 (Cont'd)

MINIMUM	TIME	U A/C	A A/C	THETA DOT	THETA	X(I) A/C	Z(I) A/C
	0.000	0.000	-4.698	-0.006	-0.018	-0.145	-87.266
MAXIMUM	14.000	10.491	-1.529	0.087	0.071	97.365	-33.500
0.000	0.000	0.000	-3.153	0.012	-0.006	0.000	-33.500
0.000	0.000	0.000	-4.479	-0.001	0.007	-0.008	-34.944
1.000	1.000	0.000	-2.127	0.066	0.017	-0.017	-36.989
1.000	1.000	0.000	-3.018	-0.004	0.041	-0.000	-38.532
2.000	2.000	0.000	-3.652	-0.012	0.032	-0.000	-40.785
3.000	3.000	1.000	-1.679	0.079	0.000	-0.015	-42.251
3.000	3.000	0.000	-3.230	-0.007	0.000	0.000	-44.000
4.000	4.000	0.000	-3.333	-0.006	0.004	1.013	-46.027
4.000	4.000	0.000	-4.301	-0.005	0.013	4.234	-48.491
5.000	5.000	0.000	-4.600	0.003	0.003	9.190	-51.263
6.000	6.000	0.000	-4.493	-0.006	0.001	14.534	-54.012
6.000	6.000	0.000	-4.516	0.003	-0.009	19.089	-56.701
7.000	7.000	0.000	-4.213	-0.003	0.006	24.998	-59.349
7.000	7.000	0.000	-4.316	0.007	-0.013	29.883	-61.917
8.000	8.000	0.000	-3.972	-0.004	0.011	35.433	-64.451
8.000	8.000	0.000	-4.177	0.003	-0.016	40.229	-66.893
9.000	9.000	0.000	-3.773	-0.009	0.014	46.002	-69.304
9.000	9.000	0.000	-4.000	0.018	-0.018	50.705	-71.628
10.000	10.000	0.000	-3.605	-0.010	0.010	56.000	-73.925
10.000	10.000	0.000	-3.811	-0.001	-0.018	61.450	-76.122
11.000	11.000	0.000	-3.453	0.013	0.015	67.501	-78.301
11.000	11.000	0.000	-3.611	-0.021	-0.018	72.327	-80.381
12.000	12.000	0.000	-3.315	0.047	0.013	78.432	-82.434
12.000	12.000	0.000	-3.006	-0.040	-0.016	83.374	-84.401
13.000	13.000	0.000	-3.164	0.000	0.010	89.472	-86.329
14.000	14.000	0.000	-2.909	-0.009	0.011	94.594	-87.266

SERIES III CASE 10



	TIME	V A/C	I A/C	THETA DOT	THETA	X(I) A/C	Z(I) A/C
MINIMUM	0.000	0.000	-0.366	-0.092	-0.024	-0.004	-59.542
MAXIMUM	14.000	21.101	-0.047	0.087	0.034	122.315	-33.000
	0.000	0.000	-1.393	0.000	-0.000	0.000	-33.000
	0.000	0.000	-0.047	0.000	-0.000	0.017	-34.185
	1.000	0.000	-2.797	-0.006	0.000	0.008	-34.894
	1.000	0.000	-1.092	0.000	0.000	0.000	-36.004
	2.000	0.000	-1.776	-0.016	0.015	0.002	-36.739
	3.000	0.000	-1.809	0.012	0.000	0.000	-39.079
	4.000	0.000	-0.025	0.007	0.021	1.296	-38.665
	5.000	0.000	-2.183	-0.003	0.024	4.639	-37.525
	6.000	0.000	-3.116	-0.042	-0.000	9.935	-1.000
	7.000	0.000	-2.870	0.042	0.011	14.964	-43.000
	8.000	0.000	-2.002	-0.047	-0.000	20.861	-41.655
	9.000	0.000	-2.012	0.077	0.000	26.838	-46.000
	10.000	0.000	-2.000	-0.000	-0.016	31.678	-47.611
	11.000	0.000	-2.123	0.087	-0.000	37.016	-48.895
	12.000	0.000	-1.904	-0.087	-0.011	42.651	-50.044
	13.000	0.000	-1.690	0.084	-0.000	48.553	-51.000
	14.000	0.000	-1.604	-0.092	-0.000	54.314	-51.951
	15.000	0.000	-1.235	0.072	-0.015	60.430	-52.696
	16.000	0.000	-0.829	-0.081	0.001	66.155	-53.332
	17.000	0.000	-0.882	0.000	-0.019	72.643	-53.806
	18.000	0.000	-0.337	-0.000	0.000	78.369	-54.174
	19.000	0.000	-0.992	0.001	0.000	85.000	-54.596
	20.000	0.000	-1.001	-0.011	0.013	93.367	-55.000
	21.000	0.000	-2.788	-0.000	0.000	104.148	-56.733
	22.000	0.000	-3.331	0.000	-0.011	116.551	-59.607
	23.000	0.000	-2.908	0.000	0.000	122.315	-59.542

SERIES III CASE 11

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APPENDIX B  
GENERALIZED AIRCRAFT EQUATIONS OF  
MOTION

B-1

GENERALIZED EQUATION  
FOR  
SIX DEGREES OF FREEDOM

X FORCE EQUATION	Page B-2
Y FORCE EQUATION	B-3
Z FORCE EQUATION	B-4
ROLLING MOMENT EQUATION	B-5
PITCHING MOMENT EQUATION	B-6
YAWING MOMENT EQUATION	

IN THE LARK-I SIMULATION, THE FOLLOWING VARIABLES ARE  
SET TO ZERO

ANGLES:

- $\phi$  ROLL ATTITUDE
- $\psi$  YAW ATTITUDE
- $\beta_s$  SIDE SLIP ANGLE
- $\Lambda_1$  LATERAL CYCLIC TILT

RATES:

- $p$  ANGULAR ROLL VELOCITY =  $\dot{\phi}$
- $r$  ANGULAR YAW VELOCITY =  $\dot{\psi}$
- $v$  LATERAL VELOCITY =  $\dot{y}$

FORCES AND MOMENTS:

- $(X)_R$  REAR ROTOR FORCE

All side forces and moments

(a) The X-Force Equation

$$X = (X)_F + (X)_R + (X)_{FUS} + (X)_W + (X)_T + (X)_{VT} + (X)_{TR} + \sum_{i=1}^n (X)_{P_i} + W \sin \phi \sin \psi$$

$$-W \cos \phi \sin \theta \cos \psi - \frac{W}{g} (\dot{u} + q\dot{w} - r\dot{v}) = 0$$

FRONT ROTOR FORCE

$$(X)_F = (L_F \cos A_{1F} - Y_F \sin A_{1F}) \sin (\alpha - \epsilon_F) - D_F \cos (\alpha - \epsilon_F) \cos \beta_s$$

$$- (L_F \sin A_{1F} + Y_F \cos A_{1F}) \sin \beta_s$$

REAR ROTOR FORCE

$$(X)_R = (L_R \cos A_{1R} + Y_R \sin A_{1R}) \sin (\alpha - \epsilon_F) - D_F \cos (\alpha - \epsilon_F) \cos \beta_s$$

$$- (L_R \sin A_{1R} - Y_R \cos A_{1R}) \sin \beta_s$$

FUSELAGE FORCE

$$(X)_{FUS} = L_{FUS} \sin (\alpha - \epsilon_{FUS}) - D_{FUS} \cos (\alpha - \epsilon_{FUS}) \cos \beta_s - Y_{FUS} \sin \beta_s + F_{LG}$$

WING FORCE

$$(X)_W = L_W \sin (\alpha - \epsilon_W) - D_W \cos (\alpha - \epsilon_W) \cos \beta_s$$

TAIL FORCE

$$(X)_T = L_T \sin (\alpha - \epsilon_T) - D_T \cos (\alpha - \epsilon_T) \cos \beta_s$$

VERTICAL TAIL FORCE

$$(X)_{VT} = -D_{VT} \cos (\alpha - \epsilon_{VT}) \cos \beta_s + L_{VT} \sin \beta_s$$

TAIL ROTOR FORCE

$$(X)_{TR} = Y_{TR} \sin (\alpha - \epsilon_{TR}) - D_{TR} \cos (\alpha - \epsilon_{TR}) \cos \beta_s - T_{TR} \sin \beta_s$$

POWER PLANT FORCE

$$(X)_{P_i} = T_{P_i} \cos i_{P_i} - W_{P_i} \sin i_{P_i}$$

(b) The Y-Force Equation

$$Y = (Y)_F + (Y)_R + (Y)_{FUS} + (Y)_W + (Y)_T + (Y)_{VT} + (Y)_{TR} + \sum_{i=1}^n (Y)_{P_i} + W \sin \theta \cos \psi \\ + W \cos \phi \sin \theta \sin \psi - \frac{W}{g} (\dot{v} + r\dot{u} - p\dot{w}) = 0$$

where

$$(Y)_F = (L_F \cos A_{1_F} - Y_F \sin A_{1_F}) \sin (\alpha - \epsilon_F) - D_F \cos (\alpha - \epsilon_F) \sin \beta_s \\ + (L_F \sin A_{1_F} + Y_F \cos A_{1_F}) \cos \beta_s$$

$$(Y)_R = (L_R \cos A_{1_R} + Y_R \sin A_{1_R}) \sin (\alpha - \epsilon_R) - D_R \cos (\alpha - \epsilon_R) \sin \beta_s \\ + (L_R \sin A_{1_R} - Y_R \cos A_{1_R}) \cos \beta_s$$

$$(Y)_{FUS} = L_{FUS} \sin (\alpha - \epsilon_{FUS}) - D_{FUS} \cos (\alpha - \epsilon_{FUS}) \sin \beta_s + Y_{FUS} \cos \beta_s$$

$$(Y)_W = L_W \sin (\alpha - \epsilon_W) - D_W \cos (\alpha - \epsilon_W) \sin \beta_s$$

$$(Y)_T = I_T \sin (\alpha - \epsilon_T) - D_T \cos (\alpha - \epsilon_T) \sin \beta_s$$

$$(Y)_{VT} = -D_{VT} \cos (\alpha - \epsilon_{VT}) \sin \beta_s - L_{VT} \cos \beta_s$$

$$(Y)_{TR} = Y_{TR} \sin (\alpha - \epsilon_{TR}) - D_{TR} \cos (\alpha - \epsilon_{TR}) \sin \beta_s + T_{TR} \cos \beta_s$$

$$(Y)_{P_i} = Y_{P_i}$$



(c) The Z-Force Equation

$$Z = (Z)_F + (Z)_R + (Z)_{FUS} + (Z)_W + (Z)_T + (Z)_{VT} + (Z)_{TR} + \sum_{i=1}^n (Z)_{P_i} + W \cos \phi \cos \theta$$

$$-\frac{W}{g} (\dot{w} + pv - qu) = 0$$

where

$$(Z)_F = -D_F \sin(\alpha - \epsilon_F) + (L_F \cos A_{1F} - Y_F \sin A_{1F}) \cos(\alpha - \epsilon_F)$$

$$(Z)_R = -D_R \sin(\alpha - \epsilon_R) + (L_R \cos A_{1R} + Y_R \sin A_{1R}) \cos(\alpha - \epsilon_R)$$

$$(Z)_{FUS} = -D_{FUS} \sin(\alpha - \epsilon_{FUS}) + L_{FUS} \cos(\alpha - \epsilon_{FUS})$$

$$(Z)_W = D_W \sin(\alpha - \epsilon_W) + L_W \cos(\alpha - \epsilon_W)$$

$$(Z)_T = -D_T \sin(\alpha - \epsilon_T) + L_T \cos(\alpha - \epsilon_T)$$

$$(Z)_{VT} = -D_V \sin(\alpha - \epsilon_{VT})$$

$$(Z)_{TR} = -D_{TR} \sin(\alpha - \epsilon_{TR}) + Y_{TR} \cos(\alpha - \epsilon_{TR})$$

$$(Z)_{P_i} = -T_{P_i} \sin i_{P_i} + N_{P_i} \cos i_{P_i}$$

where

q = PITCHING VELOCITY

p = ROLLING VELOCITY

r = YAWING VELOCITY

\theta = PITCH ATTITUDE

\phi = ROLL ATTITUDE

\psi = YAW ATTITUDE

u = VELOCITY ALONG BODY X-AXIS

v = VELOCITY ALONG BODY Y-AXIS

w = VELOCITY ALONG BODY Z-AXIS

\epsilon = ANGLE OF ATTACK OF ROTOR

A\_{1F} = CYCLIC PITCH

\beta = SIDE SLIP ANGLE

c = INTERFERENCE DUE TO DOWNWASH

X = FORCE IN DIRECTION OF BODY X-AXIS

L = LIFT FORCE NORMAL TO RELATIVE WIND

D = DRAG FORCE

Y = SIDE FORCE

T = THRUST FORCE

N = NORMAL TO THRUST FORCE

W = WEIGHT

SUBSCRIPTS

F FRONT ROTOR

R REAR ROTOR

FUS FUSELAGE

W WING

T HORIZONTAL TAIL

V VERTICAL TAIL

TR TAIL ROTOR

P POWER PLANT

LG LANDING GEAR

(d) The Rolling Moment Equation (L)

$$L = \sum_{i=1}^n (L)_i = \sum_{i=1}^n (Z)_i l_{Y_i} - (Y)_i l_{Z_i} + (L_0)_i + L_I$$

$$L = (Z)_F l_{Y_F} - (Y)_F l_{Z_F} + (Z)_R l_{Y_R} - (Y)_R l_{Z_R}$$

$$+ (Z)_W l_{Y_W} - (Y)_W l_{Z_W} + (Z)_T l_{Y_T} - (Y)_T l_{Z_T}$$

$$+ (Z)_{VT} l_{Y_{VT}} - (Y)_{VT} l_{Z_{VT}} + (Z)_{TR} l_{Y_{TR}} - (Y)_{TR} l_{Z_{TR}}$$

$$+ \sum_{i=1}^n (Z)_{P_i} l_{Y_{P_i}} - (Y)_{P_i} + Q_{P_i} + L_{FUS} + L_{HUB_F} - L_{HUB_R}$$

$$- \dot{p} I_{XX} + I_{XZ} (\dot{r} + pq) + r q (I_{YY} - I_{ZZ}) + I_{XY} (\dot{q} - rp)$$

$$+ I_{YZ} (q^2 - r^2) = 0$$

where  $i$  refers to the  $i^{th}$  aircraft component and is evaluated by letting  $i = 1, 2, 3$ , etc., or the appropriate component designation.

Subscript  $I$  refers to inertia terms. Similar notation is used in the pitching and yawing moment equations given below.

(e) The Pitching Moment Equation (N)

$$N = \sum_{i=1}^n (N)_i = \sum_{i=1}^n (X)_i l_{Z_i} + (N_0)_i + N_I$$

$$N = (X)_F l_{Z_F} - (Z)_F l_{X_F} + (X)_R l_{Z_R} - (Z)_R l_{X_R}$$

$$+(X)_W \ell_{Z_W} - (Z)_W \ell_{X_W} + (X)_T \ell_{Z_T} - (Z)_T \ell_{X_T}$$

$$+(X)_{VT} \ell_{Z_{VT}} - (Z)_{VT} \ell_{X_{VT}} + (X)_{TR} \ell_{Z_{TR}} - (Z)_{TR} \ell_{X_{TR}}$$

$$+\sum_{i=1}^n (X)_{P_i} \ell_{Z_{P_i}} - (Z)_{P_i} \ell_{X_{P_i}} + M_{P_i} + M_{FUS} + M_{HUB_F} + M_{HUB_R} + Q_{TR}$$

$$-\dot{r} I_{YY} + I_{XZ} (r^2 - p^2) - rp (I_{XX} - I_{ZZ})$$

$$+I_{XY} (\dot{p} + rq) + I_{YZ} (\dot{r} - pq) = 0$$

(f) The Yawing Moment Equation (N)

$$N = \sum_{i=1}^n (N)_i = \sum_{i=1}^n (Y)_i \ell_{X_i} - (X)_i \ell_{Y_i} + (N_O)_i + N_I$$

$$N = (Y)_F \ell_{X_F} - (X)_F \ell_{Y_F} + (Y)_R \ell_{X_R} - (X)_R \ell_{Y_R}$$

$$+(Y)_W \ell_{X_W} - (X)_W \ell_{Y_W} + (Y)_T \ell_{X_T} - (X)_T \ell_{Y_T}$$

$$+(Y)_{VT} \ell_{X_{VT}} - (X)_{VT} \ell_{Y_{VT}} + (Y)_{TR} \ell_{X_{TR}} - (X)_{TR} \ell_{Y_{TR}}$$

$$+\sum_{i=1}^n (Y)_{P_i} \ell_{X_{P_i}} - (X)_{P_i} \ell_{Y_{P_i}} + N_{FUS} + Q_F - Q_R$$

$$-\dot{r} I_{ZZ} + I_{XZ} (\dot{p} - qr) - pq (I_{YY} - I_{XX})$$

$$+I_{XY} (p^2 - q^2) + I_{YZ} (\dot{q} + pr) = 0$$

L = ROLLING MOMENT

N = PITCHING MOMENT

M = YAWING MOMENT

$\ell_{1,2}$  = A distance parallel to an axis designated by subscript 1, from c.g. to force with subscript 2

$$p = \dot{\phi} - \dot{\psi} \sin \theta$$

$$q = \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi$$

$$r = \dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta$$

$$\dot{\psi} = (q \sin \phi + r \cos \phi) \sec \theta$$

#### Transformation to Inertial Axes

$$\begin{aligned} \frac{dx'}{dt} = & u \cos \theta \cos \psi + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\ & + w(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \end{aligned}$$

$$\begin{aligned} \frac{dy'}{dt} = & u \cos \theta \sin \psi + v(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) \\ & + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \end{aligned}$$

$$\frac{dz'}{dt} = -u \sin \theta + v \sin \phi \cos \theta + w \cos \phi \cos \theta$$

APPENDIX C  
SUMMARY OF  
CHARACTERISTICS, FORCES AND MOMENTS  
OF THE  
CH-53 HELICOPTER

C-1

# SUMMARY OF CH-53 CHARACTERISTICS AND PARAMETERS

## CHARACTERISTICS

### Rotor

R	RADIUS, FT	36
b	NUMBER OF BLADES	6
$\Omega$	ROTATIONAL SPEED, RAD/SEC (NOTE 1)	21.7
$\sigma$	SOLIDITY RATIO	.1150
e	FLAPPING HINGE OFFSET, FT.	2.0
$\theta_t$	BLADE TWIST, DEG.	-6°
$\dot{L}_s$	ROTOR SHAFT INCIDENCE, DEG. (LONG.) (NOTE 2)	5°
$\gamma$	BLADE MASS FACTOR (LOCKE NO.)	12.55
$M_s$	FIRST MASS MOMENT OF BLADES, SLUG - FT	183.6
$l_{zf}$	} x,y COORDINATES OF ROTOR HEAD, FT	-7.50
$l_{xf}$		1.33
$B_{ls \max}$	} CYCLIC PITCH LIMITS, DEG. (NOTE 3)	16.25
$B_{ls \min}$		-11.0
$\theta_{o \max}$	} COLLECTIVE PITCH LIMITS	15.0
$\theta_{o \min}$		0

# CHARACTERISTICS

## TAIL

$A_t$	AREA, $FT^2$	40
$i_t$	INCIDENCE, DEG.	3.0
$A$	ASPECT RATIO	2.5
$l_{z_t}$	COORDINATES, TAIL AERODYNAMIC CENTER	-9.85
$l_{x_t}$		-25.00

## LANDING GEAR

$l_{z_{lg_n}}$	COORDINATES OF GROUND CONTACT POINT	NOSE	8.16
$l_{x_{lg_n}}$			7.08
$l_{z_{lg_r}}$		MAIN	8.16
$l_{x_{lg_m}}$			20.0

## AIRCRAFT PARAMETERS

$W$	WEIGHT, LBS	33500
$I_{yy}$	MOMENT OF INERTIA IN PITCH SLUG - $FT^2$	156000
$t_a$	AMBIENT TEMPERATURE, $^{\circ}F$	59.1
$K_e$	% ENGINE TORQUE	126.

## AFCS CONSTANTS

$K_P$	PROPORTIONAL GAIN	0.8
$K_R$	RATE GAIN	0.4
$K_S$	SUMMATION GAIN	0.0

$z_{iac}$	}	AIRCRAFT COORDINATES INERTIAL AXES, FT	33.2
$x_{iac}$			0
$\dot{z}_{iac}$	}	AIRCRAFT VELOCITY COMPONENTS	0
$\dot{x}_{iac}$			0
$\theta$		FUSELAGE PITCH, DEG	0.0
$\dot{\theta}$		PITCH RATE, RAD/SEC	0.0
T		THRUST, LBS	36000
$B_{ls}$		CYCLIC PITCH, DEGS	
$\theta_o$		COLLECTIVE PITCH, DEGS	
$\theta_t$		FUSELAGE TRIM	.014



### NOTES

- (1) Rotational speed, power on, is maintained between 21.8 rad/sec., max, and 19.4 rad/sec. min. (Ref. 5).
- (2) Inclined forward.
- (3) Positive cyclic tilts the swash plate counterclockwise.
- (4) Parameters can be changed interactively at run time.

The procedure is discussed in Chapter III. The values in the table are used unless specifically changed at the beginning of each run.

## SUMMARY OF FORCES AND MOMENTS

### LANDING GEAR (Section III-1)

#### NOSE GEAR LOAD VS STROKE ( $S_N$ )

$$F_{LG_N} = 173200 / (S_N + 4.22) \quad \text{III-4}$$

#### MAIN GEAR LOAD VS STROKE ( $S_M$ )

$$F_{LG_M} = 93000 / (S_M + 0.825) \quad \text{III-5}$$

#### DAMPING FORCES

$$F_{LG_D} = + 100 \dot{S}^2 \quad \text{III-7}$$

where  $\dot{S}$  = stroke velocity. (Subscripts: N=nose, M=main)

#### STROKE AS A FUNCTION OF AIRCRAFT/SHIP POSITION

(Both gear  $\phi$ 's are perpendicular to A/C x-axis)

$$\begin{aligned} S_N &= Z_{i_{ac}} - L_{LG} - Z_{i_{LP_N}} + l_{x_{LG_N}} \theta_f \\ S_M &= Z_{i_{ac}} - L_{LG} - Z_{i_{LP_M}} + l_{x_{LG_M}} \theta_f \end{aligned} \quad \text{III-7.1}$$

where  $Z_{i_{ac}}$  = A/C

$L_{LG}$  = Distance from cg to landing gear extended, parallel to z axis, same for both gears.

$Z_{i_{LP}}$  = Launch/landing pad contact point (inertial frame)

$l_{x_{LG}}$  = Distance in x direction, landing gear contact point to cg.

$\theta_f$  = Pitch angle, fuselage, positive nose up.

#### STROKE VELOCITY AS A FUNCTION OF A/C AND SHIP VELOCITY

$$\begin{aligned} \dot{S}_N &= \dot{Z}_{i_{LG_N}} - (\dot{Z}_{i_{ac}} + \dot{\theta}_f l_{x_{LG_N}}) \\ \dot{S}_M &= \dot{Z}_{i_{LG_M}} - (\dot{Z}_{i_{ac}} + \dot{\theta}_f l_{x_{LG_M}}) \end{aligned} \quad \text{III-7.2}$$

## ROTOR FORCES, ANGLE OF ATTACK, ENGINE POWER (Section III-2)

### 1. THRUST COEFFICIENT

$$C_T = \frac{W (T_R/W)}{\pi R^2 \rho (\Omega R)^2} \quad \text{III-8}$$

$C_T$  = Thrust coefficient

$W$  = Gross wt., lbs

$T_R$  = Rotor radius, ft.

$R$  = Rotor radius, ft.

$\rho$  = Mass density of air, slugs/ft<sup>3</sup>

$\Omega$  = Rotor rotational velocity, rad/sec<sup>2</sup>

### 2. COLLECTIVE PITCH, $\theta_0$

$$\theta_0 = \frac{D}{N}$$

$$\begin{aligned} D = & [a(t_{4,2} + t_{4,3}) - \delta_2(t_{5,6} + t_{5,7})] \\ & - \delta_1(t_{5,3} + t_{5,4}) + (at_{4,5} - \delta_2 t_{5,9}) \delta_2 \\ & + 2(at_{4,6} - \delta_2 t_{5,10}) \end{aligned} \quad \text{III-12}$$

$$\begin{aligned} N = & 2\delta_2(t_{5,8} + t_{5,9} + t_{5,10}) - 2a(t_{4,4} + t_{4,5} + t_{4,6}) \\ & + \frac{t_{3,2} + t_{3,3}}{t_{3,1}} [\delta_2(t_{5,6} + t_{5,7}) - a(t_{4,2} + t_{4,3})] \end{aligned} \quad \text{III-13}$$

$t_{i,j}$  are the coefficients from Bailey's rotor analysis, Reference 13.

$\delta_i$  are the coefficients in the section drag equation =

$$c_{d_s} = c_{d_0} + \delta_1 a_r + \delta_2 a_r^2 \quad (\text{Reference 12})$$

## Rotor Forces (Continued)

$$Q_R = C_Q (\pi R^2 \rho (\Omega R)^2) \quad \text{FT-LBS} \quad \text{III-14}$$

### 4. POWER REQUIRED

$$P_R = (Q_R \Omega) / 550 \quad \text{III-15}$$

Power available (maximum)

$$\begin{aligned} P_{A_{\max}} &= 6422 \text{ shp. } t \leq 15^\circ\text{C} \\ &= 6858 - (29.1)t \quad 15^\circ > t > 59^\circ\text{C} \quad \text{III-16} \\ &= 5140 \quad t \geq 59^\circ\text{C} \end{aligned}$$

### 5. ROTOR ANGLE OF ATTACK

$$\alpha_c = \arctan \left[ \frac{\lambda}{\mu} + \frac{C_T}{2\mu (\mu^2 + \lambda^2)^{1/2}} \right] \quad \text{III-17}$$

### 6. THE H-FORCE

$$H = \frac{-T(\sin \alpha_c - \frac{D}{L} \cos \alpha_c)}{\cos \alpha_c + \frac{D}{L} \sin \alpha_c} \quad \text{III-18}$$

$$\begin{aligned} \frac{D}{L} = \frac{c_a}{2U} &\left[ \frac{\delta_0}{\pi} t_{6,1} + \frac{\delta_1}{a} (t_{6,2}^\lambda + t_{6,3}^{0.75}) \right. \\ &\left. + \frac{\delta_2}{a} (t_{6,5}^{\lambda^2} + t_{6,5}^{\lambda^{0.75}} + t_{6,8}^{0.75}) \right] \end{aligned}$$

For these studies, the following values were used

$$\delta_0 = 0.0087$$

$$\delta_1 = -0.0216$$

$$\delta_2 = +0.400$$

$a$  is the slope of the section lift curve = 5.73

$t$  is the blade twist

# Rotor Forces (Continued)

## 3. TORQUE COEFFICIENT $C_Q$ -

$$\lambda = \frac{1}{t_{3,1}} \left[ \frac{C_T}{\sigma a} - t_{3,2} \theta_o - t_{3,3} (\theta_o + \theta_t) \right] \quad \text{III-10}$$

$$C_Q = K_1 \theta_o^2 + K_2 \theta_o + K_3 \lambda \theta_o + K_4 \lambda^2 + K_5 \lambda + K_6 \quad \text{III-9}$$

$$K_1 = (\delta_2 t_{5,8} + a t_{4,4}) + (\delta_2 t_{5,9} - a t_{4,5}) + \delta_2 t_{5,10}$$

$$K_2 = \delta_1 t_{5,3} + \delta_1 t_{5,4} + (\delta_2 t_{5,9} - a t_{4,5}) \theta_t + 2 a t_{4,6}$$

$$K_3 = (\delta_2 t_{5,6} - a t_{4,2}) + (\delta_2 t_{5,7} - a t_{4,3})$$

$$K_4 = (\delta_2 t_{5,5} - a t_{4,1})$$

$$K_5 = \delta_1 t_{5,2} + (\delta_2 t_{5,7} - a t_{4,3}) \theta_t$$

$$K_6 = \delta_o t_{5,1} + \delta_1 t_{5,4} \theta_t + \delta_2 t_{5,10} \theta_t^2$$

where  $\sigma$  = solidity ratio

$\lambda$  = inflow ratio  $[(V \sin \alpha - v)/\Omega R]$

and  $C_T, \delta_i, t_{i,j}, a, \theta_o$  and  $\theta_t$  are defined above

$C_Q$  = torque coefficient

$$= \frac{Q}{\pi R^2 \rho (\Omega R)^2 R}$$

$$L_F = T \cos \alpha_c - H \sin \alpha_c \quad \text{III-19}$$

$$D_F = H \cos \alpha_c - T \sin \alpha_c \quad \text{III-20}$$

### ROTOR HUB MOMENT Section III-3

$$M_H = K(a_1 - B_{1s}) - \left(\frac{ae}{12R}\right) \left(\frac{\dot{\theta}_f}{\Omega}\right) \left[1 - \left(\frac{e}{R}\right)^3\right] \quad \text{III-21}$$

$$K = \frac{eb\Omega^2 M_s}{2}$$

$$a_1 = t_{1,4}\lambda + t_{1,5}\theta_0 + t_{1,6}\theta_1$$

where  $e$  = flapping hinge offset, ft.

$B_{1s}$  = cyclic pitch, rad.

$a$  = section lift slope

$M_s$  = mass moment of blades, slug - ft.

$a_1$  = second Fourier flapping coefficient, (tip-path tilt) in flapping equation

$$\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi - \dots$$

(Ref. 12)

### FUSELAGE FORCES AND MOMENTS (Section III-4)

$$\text{LIFT } L_F/q = 15.0 + 487 \alpha_F \quad \text{III-24}$$

$$\text{DRAG } D_F/q = 41.696 - 11.45 \alpha_F + 4.423 \alpha_F^2 \quad \text{III-25}$$

$$\text{for } -20^\circ \leq \alpha_F \leq +20^\circ$$

$\alpha_F = \theta_F - \epsilon_{FUS} - \gamma$  = fuselage angle of attack

$\epsilon_{FUS}$  = interference of rotor on fuselage

$q$  = dynamic pressure =  $1/2 \rho v^2$

$\gamma$  = angle of climb

$$= \arctan \frac{U}{W}$$

## Fuselage Forces and Moments (Continued)

$u$  = velocity in  $x$  direction

$w$  = velocity in  $z$  direction

$$v = \sqrt{u^2 + w^2}$$

$$\text{Moment } M_F/q = -450 \quad -20^\circ \leq \alpha_F \leq -12^\circ \quad \text{III-26}$$

$$= 58.8 \alpha_F \quad -12^\circ < \alpha_F < +16^\circ \quad \text{III-27}$$

$$= 1000 \quad 16^\circ \leq \alpha_F \leq +20^\circ \quad \text{III-28}$$

## TAIL FORCES AND MOMENTS (Section III-5)

$$C_{L_T} = a(\alpha_T - \alpha_0)$$

$$\alpha_T = \theta_f + i_T - \epsilon_T - \epsilon_p$$

$$1 = C_{L_T a} = \frac{\pi AR}{1 + \sqrt{\left(\frac{\pi AR}{a_0}\right)^2 + 1}}$$

where

where

$i_T$  = Tail incidence, pos. L.E. up

$a_0$  = zero lift incidence

$\epsilon_T$  = interference angle, rotor downwash on tail (see below)

$\epsilon_p$  = change in angle of attack due to pitching velocity of fuselage

$$= \frac{\theta_f l_{x_T}}{V}$$

$l_{x_T}$  = distance of tail (1/4 pt. MAC) to cg FT.

$V$  = flight path speed =  $\sqrt{u^2 + w^2}$

$A$  = aspect ratio

0

APPENDIX D  
THE REPRESENTATION OF RANDOM SEA



## D-1 THE EQUATIONS OF SHIP MOTION

If it is assumed that the motions of an arbitrary ship are linear and harmonic, the six linear coupled differential equations of motion can be written in the following abbreviated form:

$$\sum_{k=1}^6 [\bar{M}_{jk} \ddot{\eta}_k + B_{jk} \dot{\eta}_k] = C_{nk} \eta_k + F_j e^{i\omega t}; \quad j = 1 \cdot \cdot \cdot 6 \quad D-1$$

where the six displacements are each denoted by  $\eta_k$  where the subscripts from 1 to 6 refer respectively to surge, sway, heave, roll, pitch and yaw.

It is not our intention to discuss the derivation or solution of equation D-1 or the meaning of the coefficients M, B and C. Experience with these types of equations are assumed and it is sufficient to say that M is a matrix of mass terms augmented in some fashion by the added mass of the displaced water, B is a matrix of damping constants and C is a matrix of hydrostatic restoring coefficients. For a ship with lateral symmetry under the assumption noted above, the surge, heave and sway equations may be solved independently of the other three, and the solutions of all six equations will be simple harmonic functions

$$\eta_j = A_j \cos (\omega t + \epsilon_j) \quad j=1, \dots, 6 \quad D-2$$

if the forcing function, or the equation of the wave ( $F_j e^{i\omega t}$ ), can be expressed in the simple harmonic form

$$F \cos \omega t \quad D-3$$

where  $F$  is the amplitude of the wave

$\omega$  is the encounter frequency

$A_j$  is amplitude of the ship response.

We will only be concerned with equations D-2 and D-3 and for all other matters, the reader is referred to References 1 - 3 and especially Reference 15 which is a thorough and exhaustive discussion of the derivation and solution of Equation D-1.

Equation D-2 is the sinusoidal response of the ship at a given frequency  $\omega$  to a sinusoidal wave of form D-3. In the next section, it will be shown that the real sea profile can in no realistic way be represented or approximated by the form D-3. The real sea can in no way known be represented either as instantaneous profiles

$$Z = F(x)$$

$$Z = F(x,y)$$

or as time histories

$$Z = F(x,t)$$

$$Z = F(x,y,t)$$

where the representation  $F$  is a sum, finite or infinite, of periodic functions, sinusoid or other, or polynomials such that the error

$$e = F - G(x,y,t)$$

between the representational function  $F$  and a measured profile of wave amplitudes  $G$  is either less than a preassigned value, or is a mathematical function, analytic or not, of any of the variables. This is, perhaps, the broadest statement that can be made in mathematical terms of what appears to be a truth

which can be stated in less mathematical terms as follows:  
it does not appear to be possible with the mathematical tools  
we have at hand to make a mathematical deterministic model  
of the profile of the sea surface which is a function of  
no more than a few variables, say 5 or 6. If the variables  
are limited to  $x$ ,  $y$ ,  $z$  and  $t$ , it is clearly impossible.  
Even if other variables are added such as wind strength, wind  
duration, wind direction, fetch length, boundary values of  
coasts or water depths, etc., a deterministic model is  
still impossible.

If this is true, the engineer has a problem. In nearly  
all important cases, in dynamics, the equations which he  
deals with are equations of motion or are derived from  
equations of motion, and have the form of equation D-1.  
The statements made in the prior paragraph mean that the  
expression  $F_j e^{i\omega t}$  cannot be used to represent real waves.  
The statements seem to go much further: since there is no  
deterministic way (that is a deterministic function of a few  
variables) of realistically representing a forcing function  
in equation D-1, then there is no way of comparing a solution  
of equation D-1 with what may occur in reality. For example,  
we can calculate the motions of a ship in the sinusoidal  
wave  $a_0 \cos \omega t$  we can measure the motion in a towing tank  
where such a "pure" wave shape can be generated, and compare  
the results. This is done in Reference 1 through 3. But  
we cannot compare the motion in a real sea with the calculated  
motion in "pure" waves. It was said above that the statements

in the prior paragraph "seem" to be saying this, but this, fortunately, is not altogether true. There is some truth but it is not the complete truth. In what follows, it will be shown that a statistical (as opposed to deterministic) use of the solution of Equation D-1 can be derived using a statistical representation of the real sea. The theory of the statistical representation of the random sea is rather simple. There are some serious problems in using the theory, beyond the scope of this brief survey, but which are largely concerned with how we measure wave amplitudes. It is important to remember that the statistical model is completely empirical. The deterministic model is partly empirical.

We have dwelt upon this at some length because the reconciliation of the deterministic model, exemplified by Equation D-1, of ship motions, and the statistical model of wave motions discussed below, although not difficult to understand, can lead to serious misconceptions as to what the results really mean. In most cases, the true meanings are probabilistic: "an amplitude of a given value will be exceeded only in a given time." What, then, is the meaning of choosing one amplitude rather than another? Answers to this and other questions will be attempted in the following discussion.

## D-2 The Nature of the Irregular Sea

A typical contour plot of the sea is shown in Figure D-1. The smallest circles locate the highest (or lowest) points. One should be impressed by the disturbing irregularity of the pattern, disturbing, that is, to an applied mathematician.

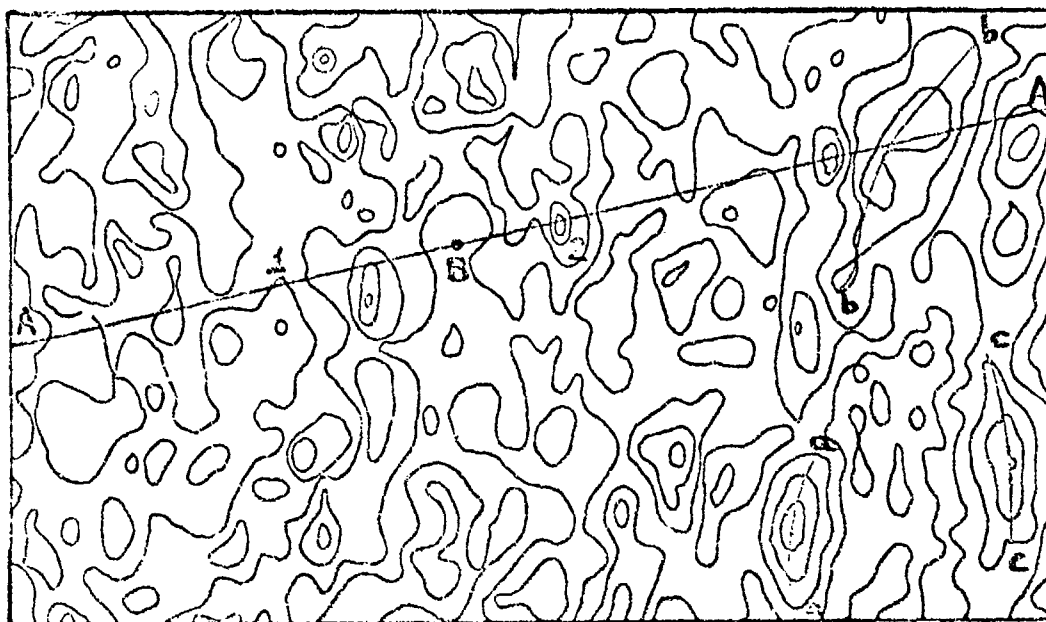


Figure D-1 Typical Contour Plot of the Sea

The picture is very messy no matter which way you hang it. If the pattern appears to have a prevailing motion (say, from left to right) we might expect the crests (and troughs) to extend indefinitely at right angles to the motion. They obviously do not. This attribute of the wave pattern is known as short crestedness, since no one crest extends very far and another appears not far away. These short crests are not aligned with each other, or of the same length or height. (Compare crests a-a, b-b, and c-c). The contours of Figure

D-1 are contemporaneous and if one were to record a multitude of such plots at succeeding instants of time and if it were possible to follow the history of each contour, it would appear as if the various crests were moving at different velocities and in different directions, although the entire pattern 'seems' to have a prevailing speed and direction. Also, crests are not very durable. They appear and disappear and their total life may not be very much longer than the apparent "period" of the wave.

If a cut is taken along some line AA, the resulting profile might look like Figure D-2.

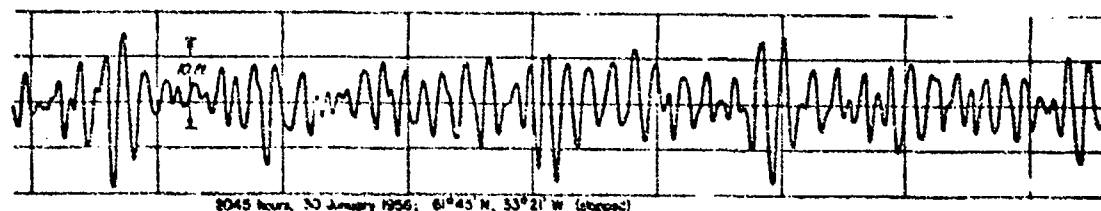


Figure D-2 Typical Wave Profile

This is the same kind of profile which might be measured by a wave height measuring device over a period of time. That is, the spatial profile of a wave pattern looks similar to that measured at a point (say B) as the waves move past.

The profile of Figure D-2 seems easy enough to understand. Let us say it was made by observing the height of the water on a staff fixed in some fashion to the ocean floor. Needless to say, this is not how the record was obtained but the remarks made below apply in any case.

Although Figure D-2 looks simple, it is enormously difficult to interpret. The "waves" are coming from many

directions although there seems to be some kind of prevailing direction. The crest lengths are all different. Point 1 moving past B is only the low end of a crest, but Point 2 is a high point. Furthermore, the crests are moving at different speeds although they give the appearance of moving in groups. A stone, for example, thrown in a still pond induces a wave train that travels toward the shore at a steady "group" velocity. But the wave lengths are shorter in the rear and longer in front. If you follow a crest, say the last, you will find that it is very quickly the next to last. As new crests appear at the rear, old crests traveling more slowly disappear at the front. This is an example of a single group speed but differing individual or "phase" speed. The phase speed,  $c$ , is related to length (for the pure deep water sinusoid) by the expression

$$c = \frac{L}{T} = \sqrt{\frac{g}{2\pi}} L = \frac{g}{2\pi} T$$

where  $T$  is the period and  $L$  is the wave length.

Figure D-3 shows the observed frequency distribution of apparent wave periods  $T$  (from Reference 16 in the form of a histogram.

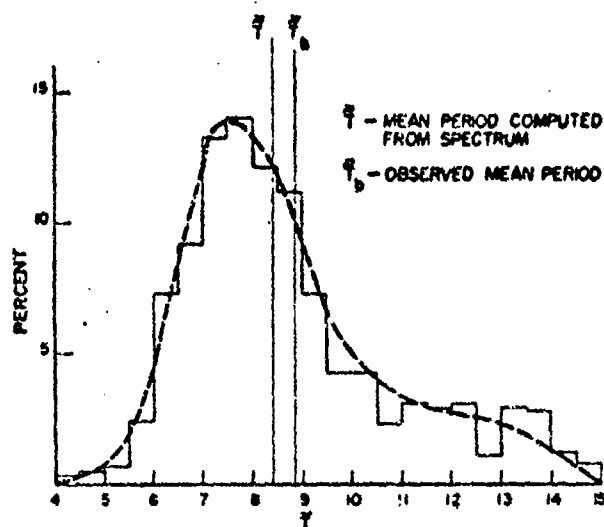


Figure D-3 Observed Frequency Distribution of Apparent Periods of Waves in a Typical Irregular Sea

If this distribution is measured at several times during a "steady" sea state, and each histogram is compiled from records of apparent period observed during a given period, and the increments  $T$  (in this case  $1/2$  sec.) are the same, the histograms will all have approximately the same shape. Although this suggests the superiority of statistical over deterministic methods, the frequency distribution does not provide enough information.

It is apparent from Figure D-3 that observed frequencies are continuously distributed from (in this case) about

$$\omega_{\max} = \frac{2\pi}{4} = 1.57 \quad (T = 4 \text{ secs.})$$

$$\omega_{\min} = \frac{2\pi}{15} = 0.42 \quad (T = 15 \text{ secs.})$$

(The actual bandwidth is somewhat larger, as we shall see. The data in Figure D-3 corresponds roughly to a sea state 5 or 6.)



A similar histogram could be derived from a single record such as Figure D-2, or a collection or ensemble of records, which furnish the distribution of wave heights. In the classical theory, however, there is no relation between wave period and wave height, except that the height-to-length ratio is limited to about  $1/7$  (Ref. 20).

Joint distributions are equally unrewarding. What appears below is an example from Reference 22.

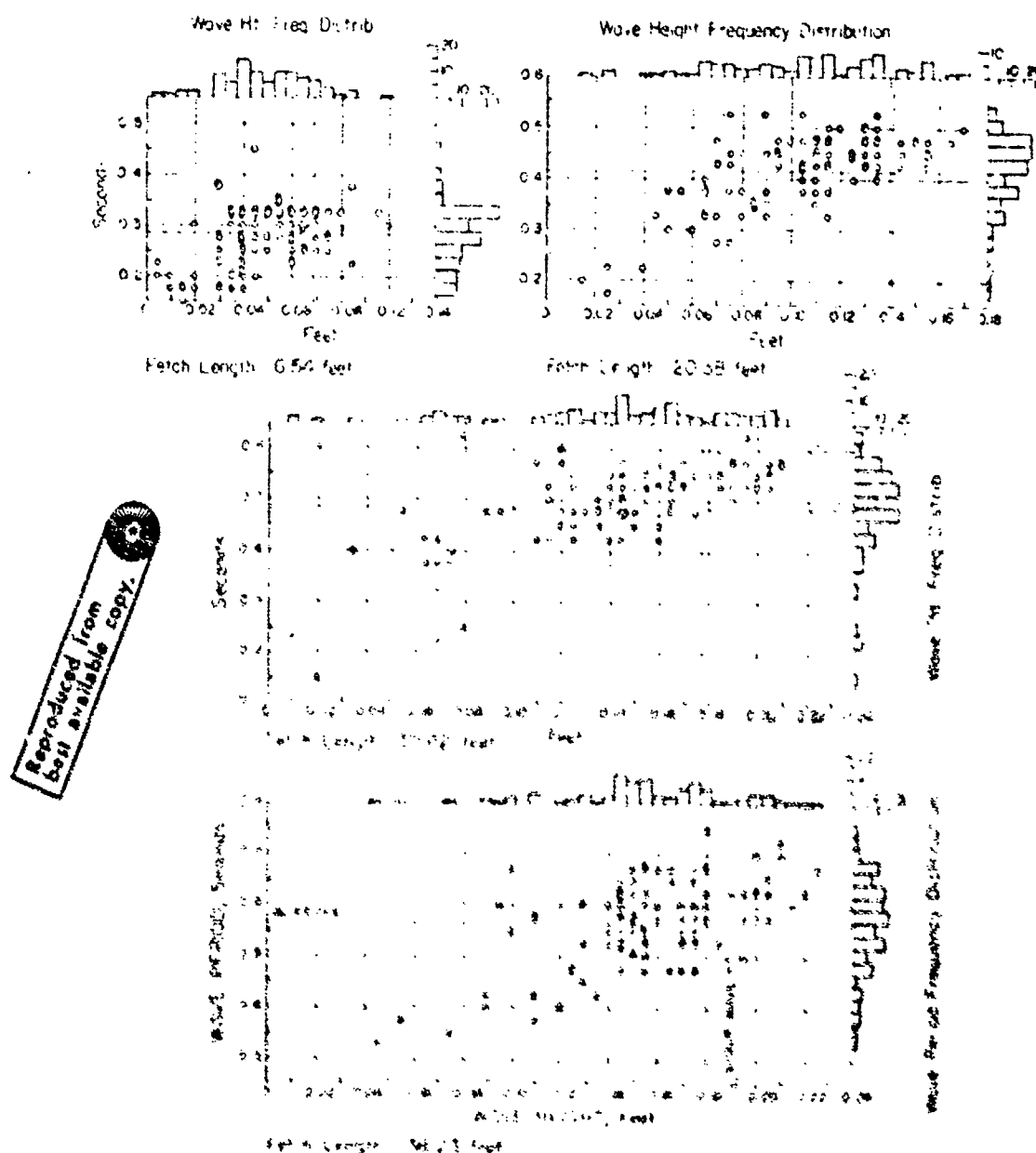


Figure D-4 Joint Frequency Distribution of Wave Period and Wave Height ( $U = 42.7$  ft/sec)

These histograms are for very small waves (2.9" max) that might appear on a pond or sheltered bay. Although the periods seem clustered about the average period, the lengths are very widely distributed about the "single wave" height.

A brief attempt has been made in this section to expose the enormous complexity of sea waves. In terms of a possible mathematical description, the sea is very irregular. Despite this, the conviction that a mathematical description is possible is greatly strengthened by the observation that over a wide area, for periods of several hours, the sea may maintain an appearance which defies description in terms of "average" or "typical" wave lengths, or periods, but nonetheless appears to be steady.

### D-3 The Specification of the Irregular Sea

Numerous mathematical models for describing a real sea state have been proposed but all have proved to be inadequate except the description in statistical terms using the concept of the continuous energy spectrum.

It is very helpful in understanding the need for a statistical description, to examine the deterministic models. It will be seen in each case that despite the deterministic character of the describing function in terms of specified amplitudes and frequencies, each method assumes information which can only be provided by statistical analysis of actual records. It should be noted that if any of the models described below were adequate, it could be used to provide wave forcing functions in equation D-1, since the solutions for these equations are for a single periodic wave of specified amplitude and frequency. The complete solution for an aperiodic wave consists of the superposition of an infinite (or in the practical case, a very large) number of regular waves, in a manner which will be discussed in the next section.

Consider the wave record of Figure D-5.

(1) This may be represented by a periodic wave whose amplitude and frequency has some significant relation to the irregular wave. It is suggested by Sverdrup and Munk (Ref. 21) that this can be done with a regular wave

$$r(t) = r_m \cos \left( \frac{2\pi t}{T} + \epsilon \right)$$

wherein  $r_m$  and  $\tau$  are the average amplitude and period of the one third highest waves. The representation is shown in Figure D-5.

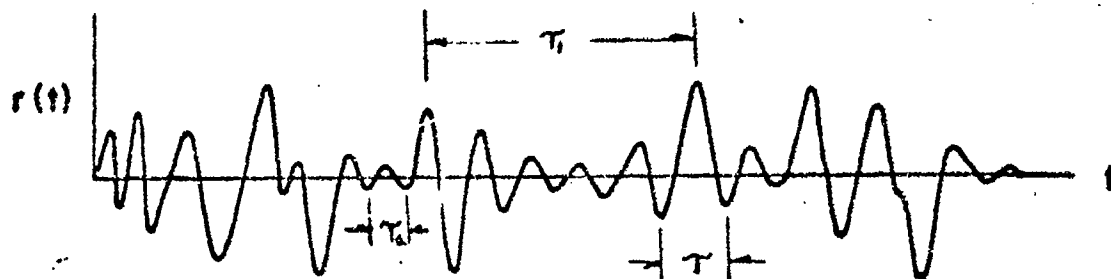
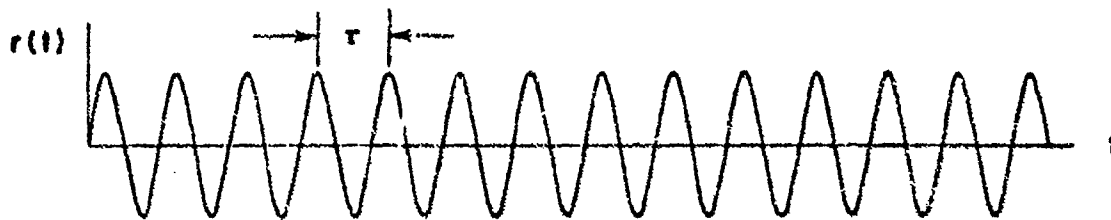


FIG. D-5.—ACTUAL WAVE RECORD



$$r(t) = r_m \cos\left(\frac{2\pi t}{\tau} + \epsilon\right) \approx r_m \cos(\omega t + \epsilon)$$

$$r(t) = r(t + \tau)$$

Figure D-5 Periodic Wave System with an Amplitude Component at a Single Spectral Frequency

A serious shortcoming of this representation is that about five sixths of the time the actual wave heights will be lower than the significant height of the idealized wave. A more serious fault lies in the selection of the period  $\tau$ . A ship acts as a narrow band filter, responding only to periods near the natural period  $\tau_n$ . If  $\tau$  is too much greater or less than  $\tau_n$ , very little motion will result. But the actual wave record of Figure D-5 contains wave components of all frequencies

and although the amplitude of a wave of period  $\tau \approx \tau_n$  may be quite small in comparison to  $r_m$ , its effect on the motion of the ship may be quite large. The idealization of Figure D-5 has filtered out these components.

(2) The irregular wave could be represented by a Fourier series with various amplitude components at many discrete spectral frequencies. This representation is shown in Figure D-6.

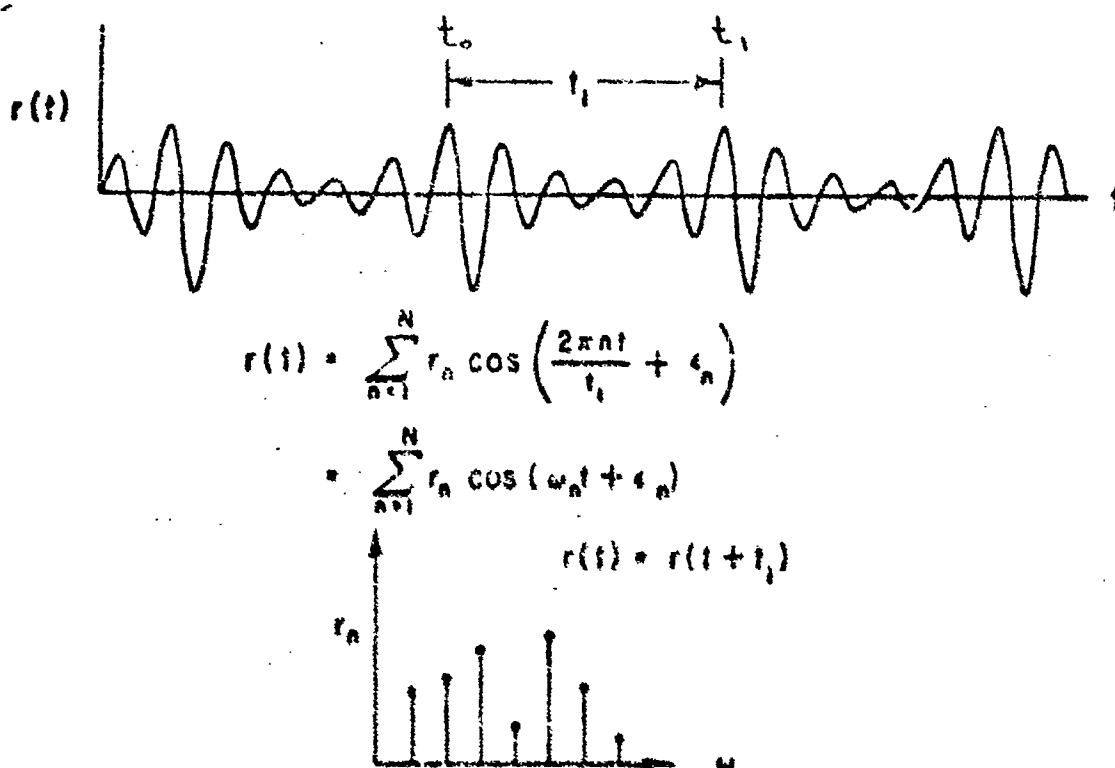


Figure D-6 Periodic Wave System with Amplitude Components at Many Discrete Spectral Frequencies

It is based on a harmonic analysis of the actual wave record of Figure D-6 for the period  $t_1$ , and for that period only is an exact representation for the limit case where  $n \rightarrow \infty$  and can

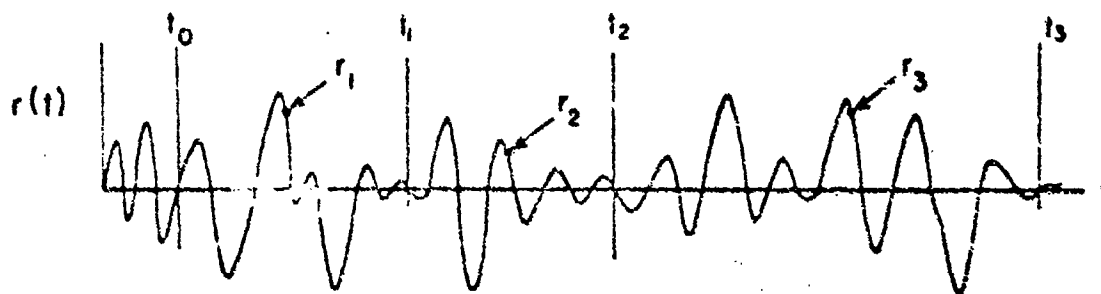
be made as exact as desired by taking  $N$  as large as necessary. But there is a severe practical limitation. In actual wave records, the amplitudes of harmonic components are negligible for periods less than 25 seconds which means that for  $n < (t_1/25)$ , all terms would drop out. The harmonics of maximum amplitude will have a period which is greater than the waves of significant (one third highest waves). The amplitudes of higher harmonics with periods less than 3 seconds would be negligible. Thus if  $t_1$  were chosen as

$t_1 =$	400 seconds	30 minutes
Neglected lower		
harmonics, $n = 1$ , to 4		70
Neglected higher		
harmonics, $n >$	30	600

The significant portion of the Fourier series for the 400 second case consists of about  $30 - 4 = 26$  terms, and for the 30 minute case of about  $600 - 70 = 530$  terms. Now it is also true that low actual waves in the Fourier series representation are caused by the phase cancellation of large numbers of harmonic components of small amplitude and large actual waves are caused by phase reinforcement of the same components. The phase angles  $\epsilon_n$  can be determined exactly by harmonic analysis for the given period  $t_0 - t_1$ , but will necessarily differ for any other time period.

(The temptation for engineers to think in terms of this model is unbelievably strong. After reading to the end of this appendix, the reader should consult Reference 23. In this paper, the author accepts the inevitability of a statistical description in terms of the energy spectrum, but reverts to a function of the form shown in Figure D-5 to approximate the spectral distribution!)

(3) A third possibility for the description of actual wave records is by use of the Fourier integral. Such a representation is shown in Figure D-7.



$$\begin{aligned}
 r(t) &= r_0(t) + r_1(t-t_1) + \dots + r_n(t-t_n) \\
 r_p(t) &= \int_{-\infty}^{\infty} a_p(\omega) \cos \omega t d\omega + \int_{-\infty}^{\infty} b_p(\omega) \sin \omega t d\omega \\
 a_p(\omega) &= \frac{1}{\pi} \int_{-\infty}^{\infty} r_p(t) \cos \omega t dt \\
 b_p(\omega) &= \frac{1}{\pi} \int_{-\infty}^{\infty} r_p(t) \sin \omega t dt
 \end{aligned}$$

Figure D-7 Aperiodic Wave System Having a Continuous Amplitude Spectrum

In this manner  $r(t)$  can be made to represent the actual wave record over any length of time chosen for analysis and this representation would be exact for the interval analyzed. It would fail, however, for time outside the interval in which the analysis was performed, since the record would either have to be defined as identically equal to zero or would remain unknown.

One disadvantage of this method is that there is no known precise procedure by which, starting with a wave record, it is possible to determine the appropriate  $a_p(\omega)$  and  $b_p(\omega)$ . Nevertheless, such a procedure is theoretically possible. The Fourier Integral method would be a convenient one for studying problems in transient response. It does not, however, lend itself to problems of the seaway because of the extremely long duration of the records to be analyzed.

The three models discussed above replace an actually measured wave profile of some definite length with a harmonic approximation. The most serious fault in each method is that the information determined in each case applies only to a specified time interval. The essential irregularity of the sea is never adequately accounted for, although the approximation to a given record could theoretically be made as close as desired. In addition there are in each case practical difficulties which have been described which seriously vitiate the usefulness of these methods.

However, for any given sea state, over appreciable lengths of time, as mentioned above, there is a "steady



state" appearance of the sea. This has led oceanographers to make use of the concept of the energy spectrum, exploiting the statistical representation of a random process in a form similar to that developed in communication theory by Tubey and Hamming, Reference 17 and Wiener, Reference 18.

The motive in discussing the three mathematical models described above is not only to exhibit their inadequacies and the need for a random statistical approach as will be described below but to clarify a loss of detail which must be accepted as a consequence of the use of the statistical method. Although in every way favorable to the "deterministic" models described above, the energy spectral method requires that the phase angles are random and independent.

#### D-4 The Statistical Representation of the Random Sea

According to this representation, the wave amplitude at a point can be represented by the integral

$$r(t) = \int_0^{\infty} \cos [\omega t + e(\omega)] \sqrt{|S_s(\omega)|^2} d\omega \quad D-5$$

It will not be necessary to use this integral (which is not an integral in the ordinary sense). This integral, which can be approximated as closely as required by a sum of difference equations, is distinguished from the previous described models by the random nature of the phase shifts  $e(\omega)$  which are equally probable and equally independent.

The most important term in equation D-5 is the spectral energy  $S_s(\omega)$ . The meaning of this term may be visualized by examining Figure D-8.

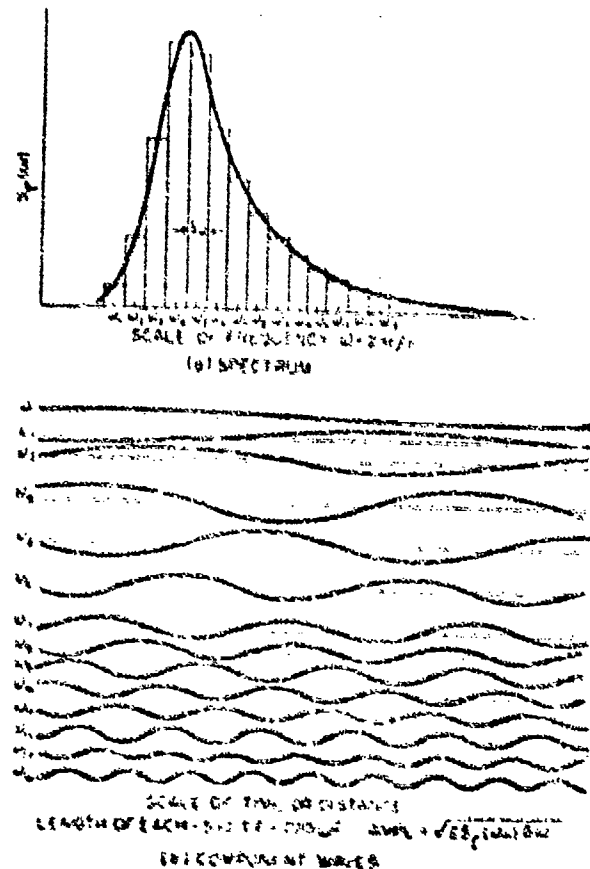


Figure D-8 Typical Energy Spectrum Showing Approximation by a Finite Sum of Components

Here it is assumed that the typical wave record  $r(t)$  is composed of a large number of sinusoidal components and that the elevation of the sea surface at any time  $t$  may be represented by the sum

$$r(t) = \lim_{\substack{\omega_n \rightarrow \infty \\ \delta\omega \rightarrow 0}} \sum \cos [\omega_n t + e(\omega_n)] [2S_s(\omega) \delta\omega]^{1/2} \quad D-6$$

The term  $S_s(\omega)$ , a continuous function of wave frequency, and is referred to as the spectral density. It comprises the totality of the statistical description of the sea state. In Figure D-8, it can be seen that  $S_s(\omega)$  is a function which for any increment  $\delta\omega$  is proportional to the total wave energy in that increment. In fact for the increment  $\delta\omega$  at the central frequency  $\omega_n$ , the total energy is

$$\rho g [S_s(\omega) \delta\omega] \quad D-7$$

where  $\rho$  is the density, and  $g$  is the acceleration constant. The total energy of the wave system represented by all the component energies in the spectrum is given by the integral

$$\rho g \int_0^\infty S_s(\omega) d\omega$$

This integral, which represents the area under the spectral curve of Figure D-2, is usually called  $E$  and hence the total energy is  $\rho g E$ .  $S_s$  has the units of (length<sup>2</sup> x time).

While the energy spectrum  $S_s(\omega)$  has the appearance of an amplitude spectrum, it is different in a very significant way which explains its unique ability to characterize the random

sea. The potential energy of a single sinusoidal wave (per unit length of crest) can be represented by

$$\frac{1}{2} \rho g r^2$$

where  $r$  is the wave amplitude. It follows, then that at a given central frequency  $\omega_n$ , a fictitious wave having the same energy as represented by equation D-7 would have the height

$$r_n = [2S_s(\omega) d\omega]^{1/2}$$

The spectral density  $S_s(\omega)$ , however, represents the cumulative effect of all measurable amplitudes in the increment  $d\omega$  centered at  $\omega_n$ .

$S_s(\omega)$  can be readily determined from amplitude time histories such as Figure D-3. The energy spectrum is in fact the Fourier transform of the autocorrelation function of a given amplitude time history. Thus

$$S_s(\omega) = \frac{1}{2\pi} \int_0^{\infty} r(t) r(t-\tau) e^{-i\omega\tau} d\tau \quad D-8$$

where  $\tau$  is a continuously varying time lag.

There are two cardinal points to keep in mind in assessing the value of the spectral density method. The function  $r(t)$  given by equations D-5 and D-6, although it represents a statistically valid profile will not be used in the subsequent development of ship motions. Only the spectral density  $S_s(\omega)$  is required. As will be shown in the next section, this approach provides a statistical description of ship

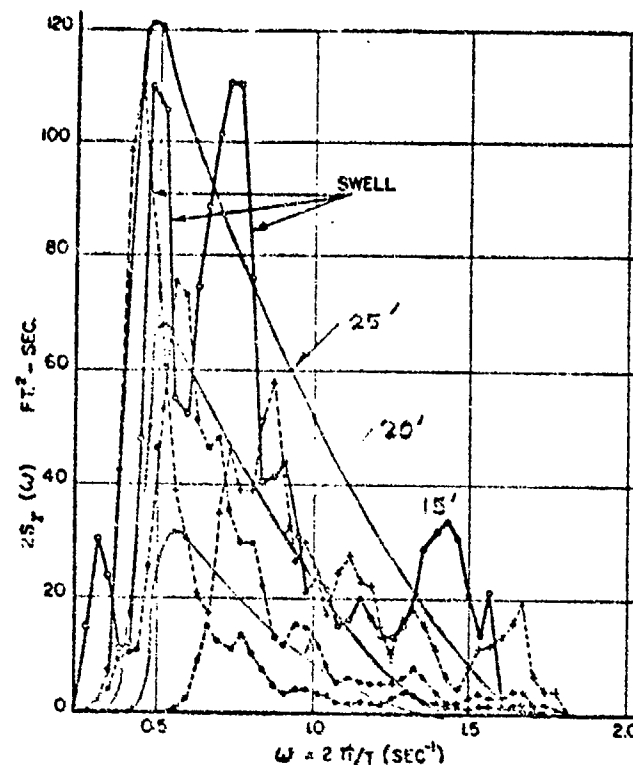


Figure D-9 Typical Sea Spectra

motions in the frequency domain. The implication of this, which has been alluded to above, is that this approach discards all information which relates to the relative phase of the various frequency components. The effect of this will be to introduce a degree of indeterminacy in the equations of aircraft motion whose importance will be very hard to assess.

## D-5 Ship Motions in an Irregular Sea

The solutions of equation D-1 for each frequency are linear solutions of the form D-2. If we knew the harmonic components of each frequency band  $\delta_\omega$  in Figure D-9, specifically the amplitude of each component  $\omega_i$  then we could combine the motions by the principal of superposition and determine a total motion which applies to all forcing functions present in the sea pattern. This, however, requires that we must use one of the three deterministic models shown in Figures D-5, D-6, and D-7. These are, in fact, the deterministic models which cannot be used for the reasons discussed in the previous section. The use of the sea energy spectrum of one of the experimental forms shown in Figure D-9 requires that we use the statistical information which these spectra contain in a manner which is within the applicability of the principal of superposition.

The statistical sea energy spectral data is used with the solutions of equation D-1 in the following way: A spectrum of ship response amplitudes as a function of (encounter) frequency is determined by the use of the energy spectrum  $S_g(\omega)$  and the ship response operator  $r(\omega)$  by the formula

$$S_R(\omega) = r(\omega) S_g(\omega) \quad \text{D-9}$$

The ship response operation is defined as the squared value of the response  $\eta_j(\omega)/\lambda_j(\omega)$  (see equation D-2)

$$r(\omega) = [\eta_j(\omega)/\Lambda_j(\omega)]^2$$

for each significant value of  $\omega$ . Using the symbol  $[R(\omega)]^2$  for  $r(\omega)$ , equation D-9 may be written

$$S_R(\omega) = [R(\omega)]^2 \cdot S_S(\omega)$$

a form in which it often appears in the literature. (See, for example, p. 31 of Reference 2, and page II-8, this report.)

The individual results in the calculation of equation D-9 are illustrated in Figure D-10 and will be explained in some detail below. Figure D-10 shows the application of the principle of superposition for sea spectrum at a fixed point (Figure D-10(a)) for two typical ships: a 250' ship moving at 7.97 knots (Froude number  $Fr = 0.15$ ) and a 500' ship moving at 11.27 knots ( $Fr = 0.15$ ). In terms of the frequency of encounter,  $\omega$ , the energy spectra are shifted to the right by the transformation

$$\omega = \omega_0 \left(1 + \frac{\omega_0 U}{g}\right)$$

where  $\omega_0$  is the wave frequency and  $U$  is the ship speed. In addition, as explained in Reference 3, the spectral amplitudes are transformed by multiplying each amplitude by the Jacobean of the transformation:

$$\frac{1}{\left(1 + \frac{\omega_0}{g} U\right)^{1/2}}$$

Naturally, the total areas under the original and transformed curves are identical. In Figure D-10b the curves are smoothed curves drawn through a large collection of individual solutions to equation D-1. For example, at the center

frequency  $\omega_n = 0.8$ , Figure D-10b shows that the ship response in pitch, scaled by the wave amplitude  $A_5$  is about (for the 250' ship),

$$(\theta/5)^2 = 2 \times 10^{-4} \quad (= \eta_5/A_5)^2 \quad (\text{Continued page D-25})$$

Notes for Figure D-10:

- (1) The units of each of the sets of curves are as follows:

Fig. 9(a) = Units of  $S_s(\omega)$  are  $FT^2 - SEC$ .

Fig. 9(b) = Units of  $\theta/5$  are  $(FT^2 - SEC^2)^{-1}$

Fig. 9(c) = Units of  $S_\theta(\omega)$  are  $(SEC)^{-1}$

(2) The spectrum  $S_s(\omega)$  may either be determined numerically from an actual wave (one-dimensional) record such as that in Figure D-2 or analytically by a formula such as the Pierson-Moskowitz equation cited on p. 25, Reference 2, or the Neuman Spectra (equation 1.28, Reference 7). In the former case, the auto correlation of the wave record is obtained

$$\phi_z(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T z(t) z(t + \tau) dt$$

The Fourier transform of  $\phi_z(\tau)$  is the power spectrum for the variable  $z$ .

$$S_z(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \phi_z(\tau) e^{-j\omega\tau} d\tau$$

In the latter case, the amplitude of the sea spectrum is expressed for any given frequency as a function of some parameter, for example significant wave height  $H_{1/3}$  (in the Pierson Moskowitz equation).

$$S_s(\omega) = (A/\omega^3) \exp(-B/\omega^4)$$



Notes for Figure D-10 - Continued

where  $A = (8 \times 10^{-2}) g^2$

$$B = (33.56) h_{1/3}$$

$\omega$  = wave frequency

or (for the Neuman spectrum) as a function of wind velocity

$$S_s(\omega) = (c/\omega^6) \exp(-2g^2/U^2\omega^2) \cos^2 x_m$$

$$c = 32.9 \text{ FT} - \text{SEC}^{-5}$$

$U$  = wind velocity

$x_m$  = angle between wind and wave patterns.

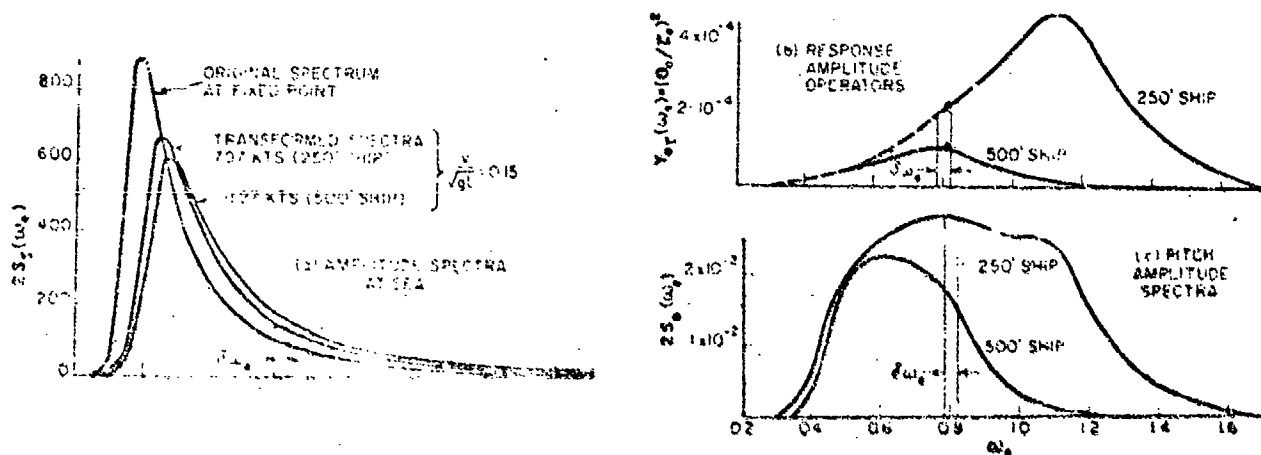


Figure D-10 Typical Pitch Responses

where  $\theta$  (and  $\eta_5$ ) is the pitch in radians and  $\lambda$  (and  $\eta_3$ ) are the wave amplitudes, and about 1/2 that for the 500' ship. The spectrum of Figure D-10b is the ship response amplitude operator referred to in equation D-9 above as  $r(\omega)$ . (Note: in the literature the various ship response amplitude operators are defined and non-dimensionalized in various ways. For example, in Reference 2, the response amplitude operator (RAO) in pitch is scaled by  $2\lambda$  wave amplitude/ship length:

$\theta L/2S$ ,  $L$  is the ship's length and  $\zeta$  is the wave amplitude. In the case of that reference, equation 5 is used. The difference in the curves for the 250' and 500' ships are very significant. They illustrate the general principals that apart from hull shape and weight distribution, that wave components of only certain frequencies have significant effects on ship motions. In general:

(a) Wave components of about  $3/4$  ship length or above have appreciable effects in pitching motions in head seas,

(b) The most severe motions result at about the ship's natural pitching frequencies. The same kind of statements can be made about all ship motions components.

The total pitch ship response in a particular sea state represented by the sea state spectrum of Figure D-10a is determined by multiplying ordinate by ordinate the spectra of Figure D-10a and D-10b to obtain the spectrum of Figure D-10c. Again, it will be noted that there is a significant change in the shape of the spectra. For example, the peak amplitude of the RAO for the 250' ship is at about a frequency of 1.2, whereas the pitch amplitude spectra for the same ship is relatively flat between frequencies of 0.6 and 1.7 radians/second. The meaning of the shifting peak amplitudes in the three diagrams should be clearly understood.

(a) In the sea spectrum Figure D-10a the peak amplitude of the original spectrum is at about  $\omega_0 = 0.4$ . The forward speeds of the two ships shifts the frequency of encounter slightly to the right since the ships are directed

into head-on seas.

(b) The smooth curves of Figure D-10b represents an interpolation through an ensemble of solutions to the pitch equation of equation D-1, wherein the pitch  $\theta$  has been scaled by the wave amplitude. That is

$$(\theta/\zeta)^2 = (\eta_5/a_0)^2$$

where  $a_0$  is the amplitude of the exciting wave. Each point on the smoothed RAO curve of Figure D-10b represents a single independent solution of equation D-1 (in this case the pitch equation). That is to say, the point on the faired curve  $(\theta/\zeta)^2$  quoted above at an encounter frequency of 0.8 (for the 250' ship) is the  $\eta_5$  (or pitch) solution to equation D-1 for the (unit) wave forcing function of

$$z/a_0 = \cos(0.8t) \quad (\text{Head-on})$$

Likewise, the maximum amplitude for the 250' ship which occurs at an encounter frequency of about  $\omega = 1.1$  radians/second is the single particular  $\eta_5$  solution for the particular (unit) wave

$$z/a_0 = \cos(1.10t)$$

For each motion equation D-1 must be solved to determine separately each point on the RAO. In Reference 2, for example, solutions for pitch and heave are calculated at increments of the encounter frequency of 0.2 from 1.0 to 6.4.

(c) The pitch amplitude spectra Figure D-10c is for each frequency, the product of the transformed sea spectra (in terms of encounter frequency) and the RAO of Figure D-10b. However, what is important to realize is that the pitch

spectrum ( $s_{\theta}(\omega)$ ) of Figure D-10c is itself a spectrum of exactly the same statistical nature as Figure D-10a. The spectral curve tells us nothing directly about the expected amplitudes of pitching and nothing whatsoever concerning the frequencies at which these expected amplitudes might occur. Statistically, the expected amplitudes are related to the total area under the pitch amplitude spectra, Figure D-10c. Thus for the total energy

$$E_{\theta} = \int_0^{\infty} S_{\theta}(\omega) d\omega = 1/2 \int_0^{\infty} 2 S_{\theta}(\omega) d\omega$$

statistical theory states that for a typical "Rayleigh" distribution, the average amplitude  $\theta_n$ , "significant" (or average of 1/3 largest)  $\theta_{1/3}$ , and average of the 1/10<sup>th</sup> highest amplitudes  $\theta_{1/10}$ , are given by the expressions

$$\theta_{av} = 1.253 \sqrt{E_{\theta}}$$

$$\theta_{1/3} = 2.000 \sqrt{E_{\theta}}$$

$$\theta_{1/10} = 2.546 \sqrt{E_{\theta}}$$

Thus, in the case quoted in Figure D-10, the total "energy" in the curve for the 350' ship from  $\omega_{\min}$  to  $\omega_{\max}$  is about

$$2.5 \times 10^{-2} \text{ (SEC}^2\text{-FT}^2\text{)}^{-1} \text{ thus,}$$

$$\theta_{av} = \pm 1.8^{\circ}$$

$$\theta_{1/3} = \pm 2.9^{\circ}$$

$$\theta_{1/10} = \pm 3.7^{\circ}$$

Although this fact has been referred to with varying degrees of emphasis several times in the foregoing discussion it is considered very important in view of what will follow to enlarge upon it again:

Although the various "statistical" amplitudes referred to above ("average", "significant" or "1/10<sup>th</sup> highest") are functions of the total area of the energy curve, this method provides no information to associate these amplitudes with any specific frequency. Although our mathematical intuition assures us that there is no other way for it to be, it is nonetheless a feeling of somehow being cheated that overcomes us when we review the three sets of curves in Figure D-10. The curve of Figure D-10b is composed of a faired interpolation of points, each of which as a solution of one of the equations of the set equation D-1 is a solution which combines both amplitude and frequency (as well as phase lag) information. For example, as quoted above, at an encounter frequency of 0.8, where  $(\theta/L)^2 = 2 \times 10^{-4}$ , this ship pitches about

$$\theta = 0.815^\circ/\text{FT of wave}$$

That is to say that for a 2' wave of the form

$$z = 2 \cos 0.8t$$

The ship would pitch according to the expression

$$\eta_5 = 0.815 (\cos 0.8t + \epsilon_5)$$

(where  $\epsilon_5$  although determined by the solution of equation D-2 is not included in the data of Figure D-10b). However, in the generation of Figure D-10c, both the frequency and phase information of solution D-2 is discarded, and amplitudes of given dimensions (such as "average" or "significant" or "average of 1/10<sup>th</sup> highest") or the probability of an amplitude of any given height is determined from the total

integrated area of the  $S_0(\omega)$  curve. For example, statistical theory for this type of distribution (the "Rayleigh" distribution) states that the probability that the amplitude  $\theta$ , a given response will exceed an amplitude  $\theta$ , given height  $\theta_1$ , is

$$P(\theta > \theta_1) = \exp(-\theta_1^2/2E)$$

Since the "significant" amplitude "average of the 1/3rd highest) for a Rayleigh solution is

$$\theta_{1/3} = 2.00 E_\theta$$

where

$$E = \int_0^\infty S_0(\omega) d\omega$$

for any sufficiently large number  $N$  such that  $N = P(\theta_N > \theta_{1/3})^{-1}$  the most probable largest pitching amplitude  $\theta_N$  is

$$\theta_N = \theta_{1/3} \ln N / 2$$

No similar information is provided from Figure D-10c for the most probable frequency or range of frequencies for which a given amplitude may occur.

APPENDIX E

COMPILATION OF HARRIER DATA

## COMPILATION OF HARRIER DATA

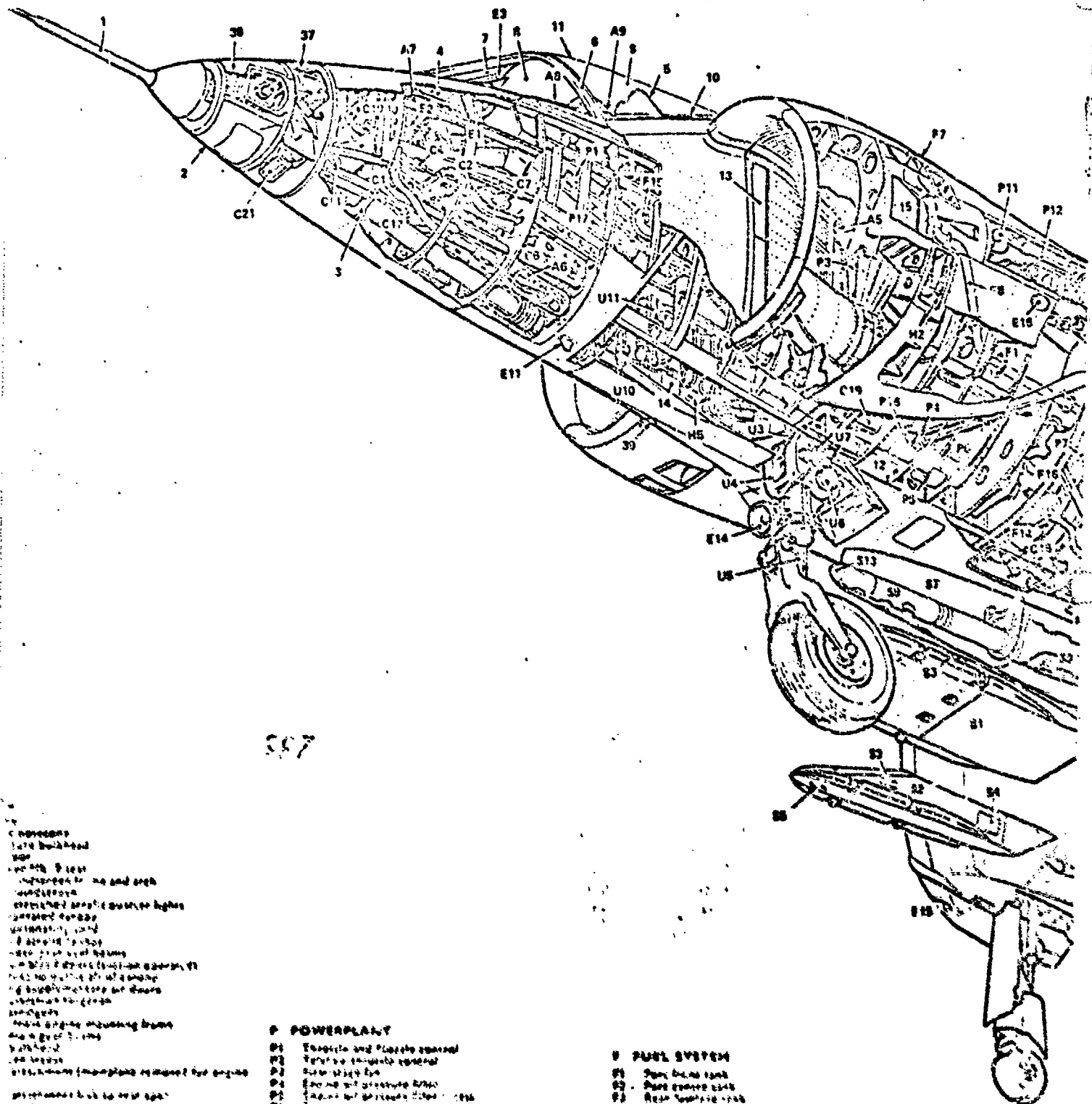
The data on the Harrier GR Mark 1 aircraft which follows was received by CADCOM after the conclusion of the project described in this report. It was furnished by Dr. Bernard H. Carson of the Office of Naval Research, London.

CADCOM includes it herein strictly for information purposes and makes no representations about its accuracy or completeness.



# **N HYDRAULICS**

- M1 Nitrogen bottle w/ emergency operation
- M2 General service system hand pump and pressure gauge
- M3 Ground servicing point No. 1 system
- M4 Reservoir No. 1 system
- M5 Accumulator, nose wheel steering



387

1. Nose  
 2. Late fuselage  
 3. Wing  
 4. Wing  
 5. Wing  
 6. Wing  
 7. Wing  
 8. Wing  
 9. Wing  
 10. Wing  
 11. Wing  
 12. Wing  
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 100. Wing

## **P POWERPLANT**

- P1 Torque and pressure control
- P2 Torque and pressure control
- P3 Torque and pressure control
- P4 Torque and pressure control
- P5 Torque and pressure control
- P6 Torque and pressure control
- P7 Torque and pressure control
- P8 Torque and pressure control
- P9 Torque and pressure control
- P10 Torque and pressure control
- P11 Torque and pressure control
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- P98 Torque and pressure control
- P99 Torque and pressure control
- P100 Torque and pressure control

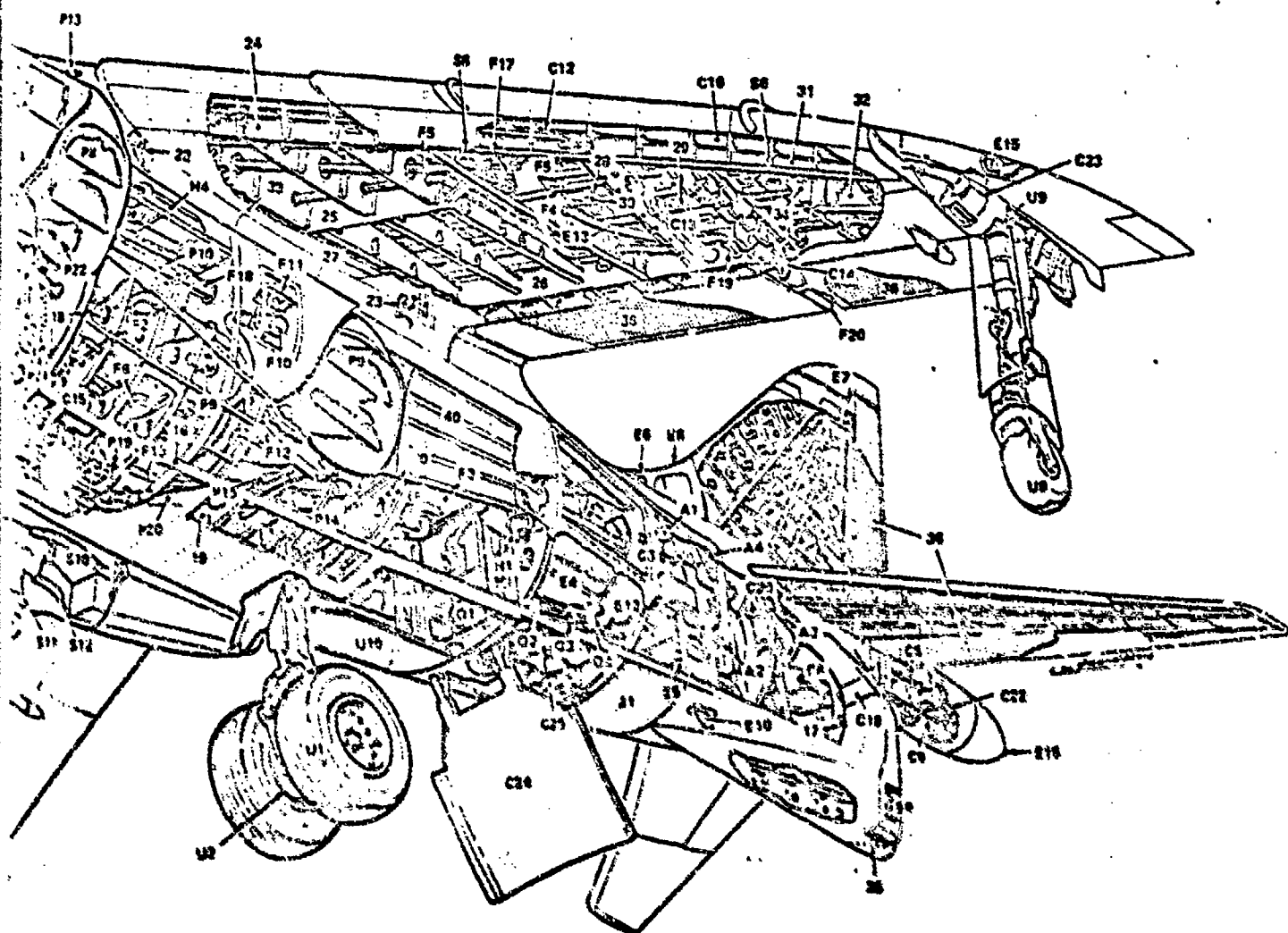
## **Q FUEL SYSTEM**

- Q1 Fuel tank
- Q2 Fuel tank
- Q3 Fuel tank
- Q4 Fuel tank
- Q5 Fuel tank
- Q6 Fuel tank
- Q7 Fuel tank
- Q8 Fuel tank
- Q9 Fuel tank
- Q10 Fuel tank
- Q11 Fuel tank
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- Q98 Fuel tank
- Q99 Fuel tank
- Q100 Fuel tank

- U1 Rigid live-safe mounted mainwheel
- U2 Multi-disc brakes
- U3 Retraction jack
- U4 Pre-shortening nose-gear leg
- U5 Liquid Spring unit
- U6 Steering motor plus or minus 45 deg. (170 deg. disengaged)
- U7 Steering control valve
- U8 Outrigger wheels (100 deg. castor inwards only)
- U9 Leg pivot
- U10 Friction: door
- U11 Door jacks

- E1 Nav/attach display
- E2 Nav/attach computer
- E3 Sight and head-up display
- E4 Shockproof equipment tray with integral cooling duct carrier Tacan, IFF, voice recorder, temp amplifier. Lower tray air data, nav/attach equipment
- E5 Transformer-rectifier unit
- E6 HF Tuner
- E7 VHF serial
- E8 HF notch serial
- E9 IFF notch serial
- E10 UHF stand-by serial
- E11 Tacan
- E12 Batteries, 28v 25Ah
- E13 Cable ducts
- E14 Landing lamp
- E15 Nav lights
- E16 Ground intercom

- C1 Rudder input from pedals
- C2 Rudder quadrant
- C3 Rudder cable tensioner
- C4 Rudder control unit
- C5 Linkage from rudder to yaw-reaction nozzle
- C6 Tailplane control input from columns
- C7 Tailplane quadrants
- C8 Tandem tailplane jack
- C9 Linkage to pitch reaction nozzle
- C10 Pitch spring foot
- C11 Pitch C-heel
- C12 Aileron control rod
- C13 Tandem aileron jack with auto-stabiliser
- C14 Aileron offset hinge
- C15 Reaction control bleed air supply shut-off (open after 15 deg. nozzle rotation)
- C16 Duct to roll-reaction valve
- C17 Duct to pitch-reaction valve
- C18 Duct to pitch and yaw-reaction valve
- C19 Control cable and pipe run duct
- C20 Rudder and tailplane cables
- C21 Pitch reaction valve
- C22 Pitch and yaw reaction valve
- C23 Roll reaction valve
- C24 Airbrake
- C25 Airbrake jacks



#### 8 STORES

- S1 20 L. 1000 lb. stores carrier
- S2 Gunner station
- S3 External store/weapon loading
- S4 Anti-landing store base jacks
- S5 Store holding mechanism
- S6 Store mounting bracket
- S7 Store gun base
- S8 Store adaptor gun
- S9 Store suspension points
- S10 Store
- S11 Gun gunner
- S12 Gun gunner
- S13 Store gun base

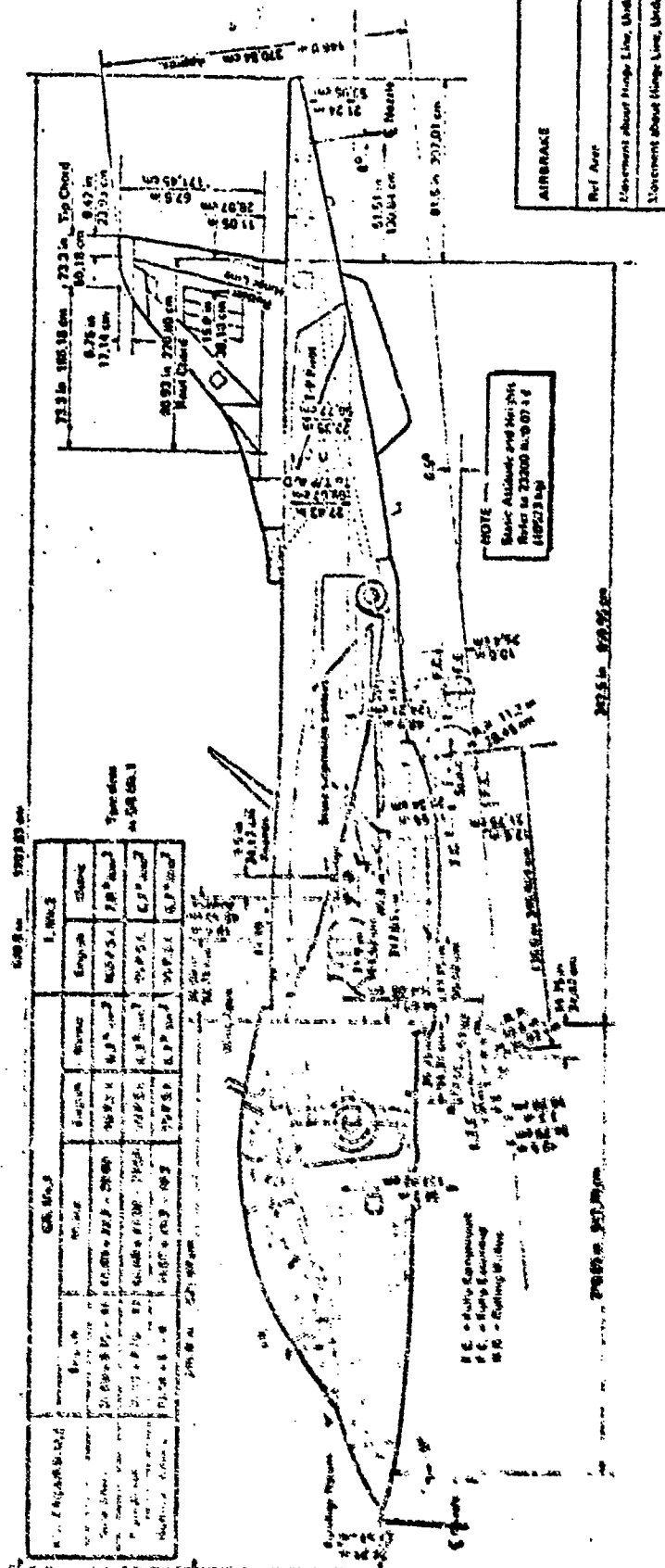
#### A AIR SYSTEM

- A1 Air intake
- A2 Compressor
- A3 Air filter
- A4 Air filter
- A5 Air filter
- A6 Air filter
- A7 Air filter
- A8 Air filter
- A9 Air filter
- A10 Air filter
- A11 Air filter
- A12 Air filter
- A13 Air filter
- A14 Air filter
- A15 Air filter
- A16 Air filter
- A17 Air filter
- A18 Air filter
- A19 Air filter
- A20 Air filter

#### 9 OXYGEN

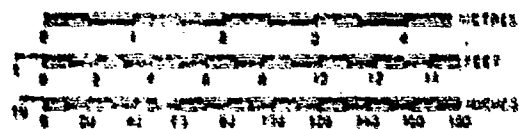
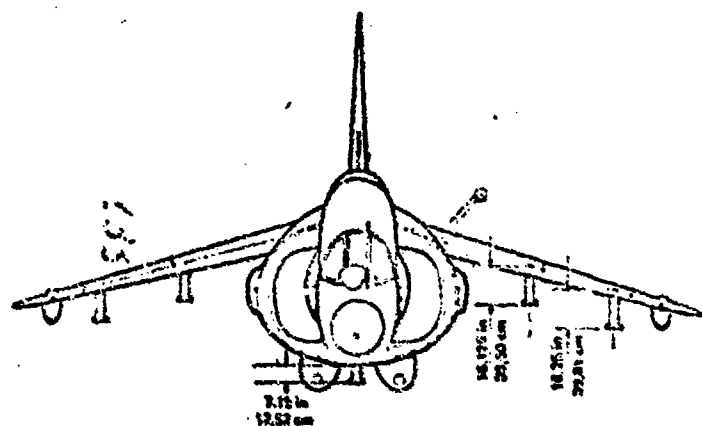
- O1 Fuel-line line connection
- O2 Fuel-line line
- O3 Fuel-line line
- O4 Fuel-line line

Hawker Siddeley  
*Harrier*



AIRRAKE	1942-1943	English	Malay
Ref. Acpt	8.5 x 7	0.75 x 2.5	650
Experiment about Hinge Line, Undercarriage Lift			250
Experiment about Hinge Line, Undercarriage Down			

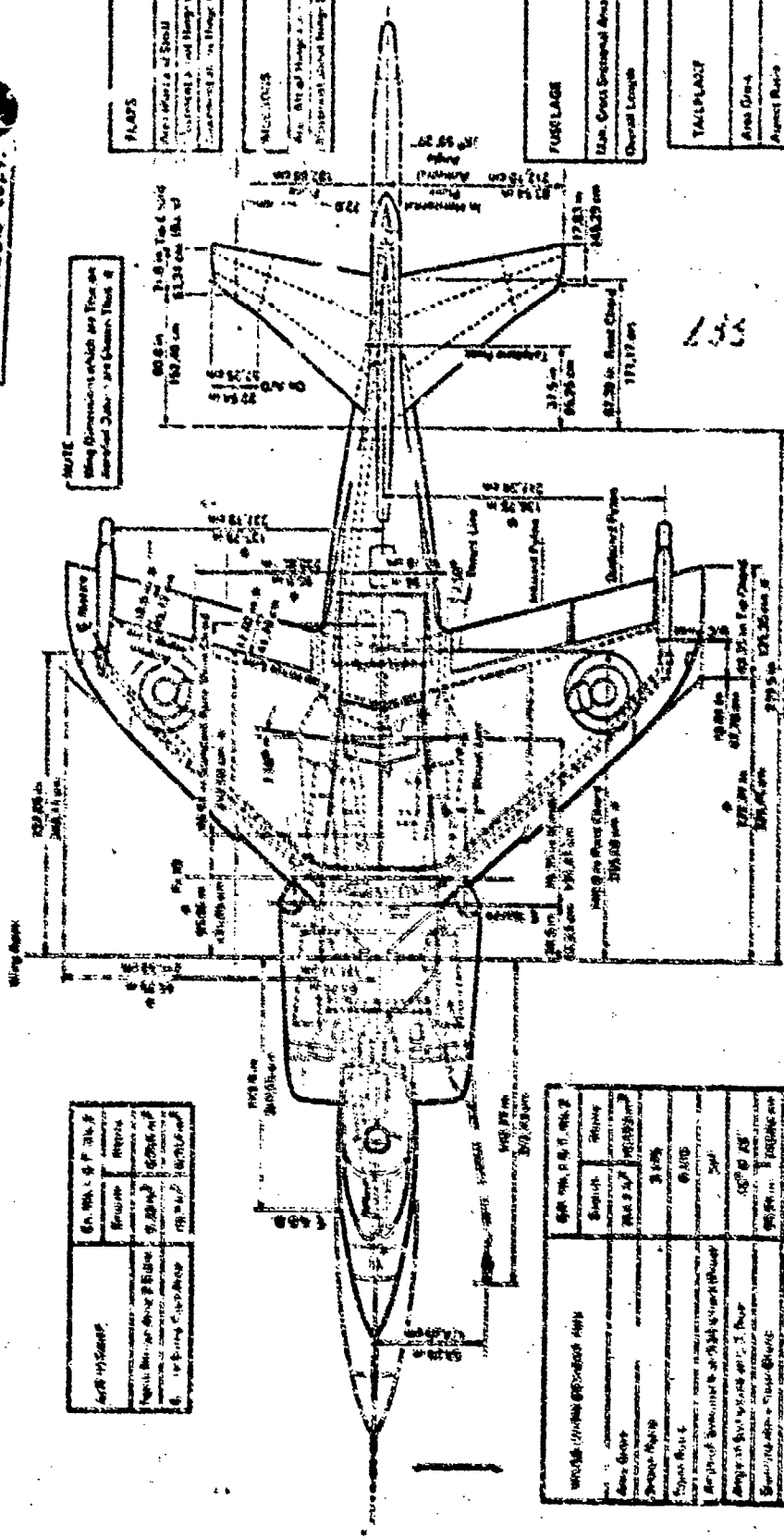




WING AND BLADES	WING		BLADE	
	Length	Area	Length	Area
Wing Area (including Blading)	24.82 m <sup>2</sup>	2.80 m <sup>2</sup>	24.82 m <sup>2</sup>	2.80 m <sup>2</sup>
Wing Area (excluding Blading)	12.41 m <sup>2</sup>	1.40 m <sup>2</sup>	12.41 m <sup>2</sup>	1.40 m <sup>2</sup>
Area of Blade at 1/2 Chord	45° 22'	45° 22'	45° 22'	45° 22'
Area of Blade at 3/4 Chord	45° 22'	45° 22'	45° 22'	45° 22'
Wing Area at 1/2 Chord	2.402 m <sup>2</sup>	2.80 m <sup>2</sup>	2.402 m <sup>2</sup>	2.80 m <sup>2</sup>
Wing Area at 3/4 Chord	2.402 m <sup>2</sup>	2.80 m <sup>2</sup>	2.402 m <sup>2</sup>	2.80 m <sup>2</sup>



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WING AREA	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

WING AREA	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

WING AREA	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

FUSELAGE	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

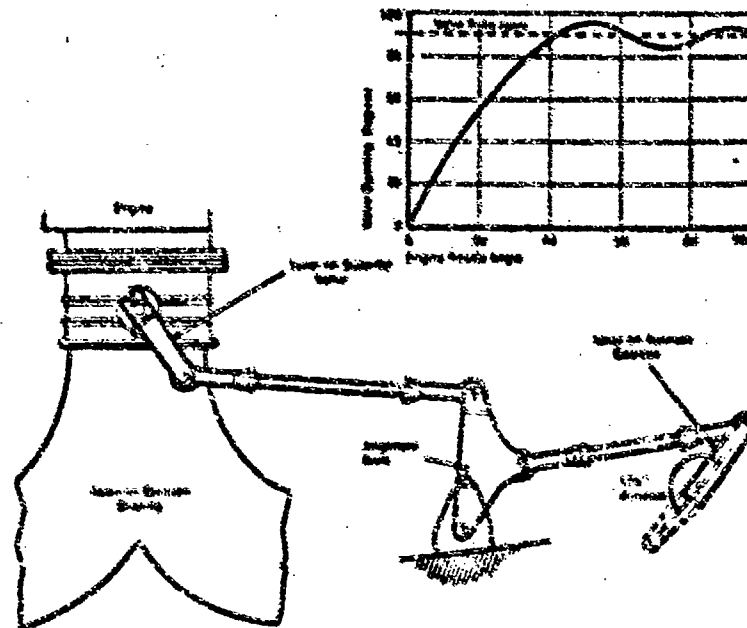
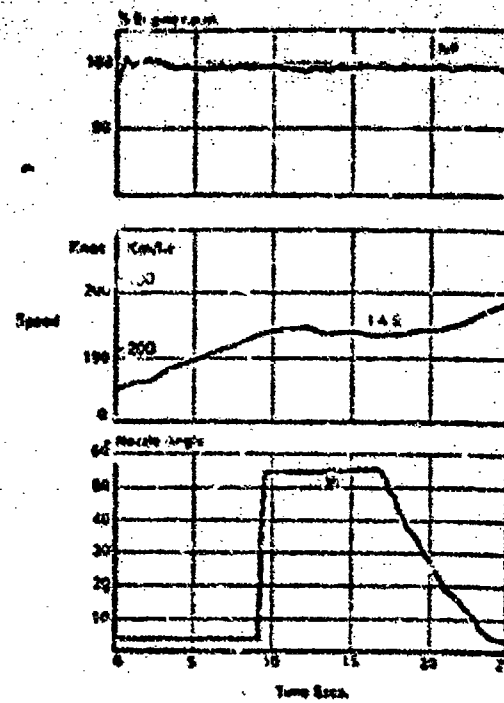
FUSELAGE	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

FUSELAGE	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

TAILPLANE	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

METRES	
Span	Area
100.00 ft	1,000.00 sq ft
100.00 ft	1,000.00 sq ft

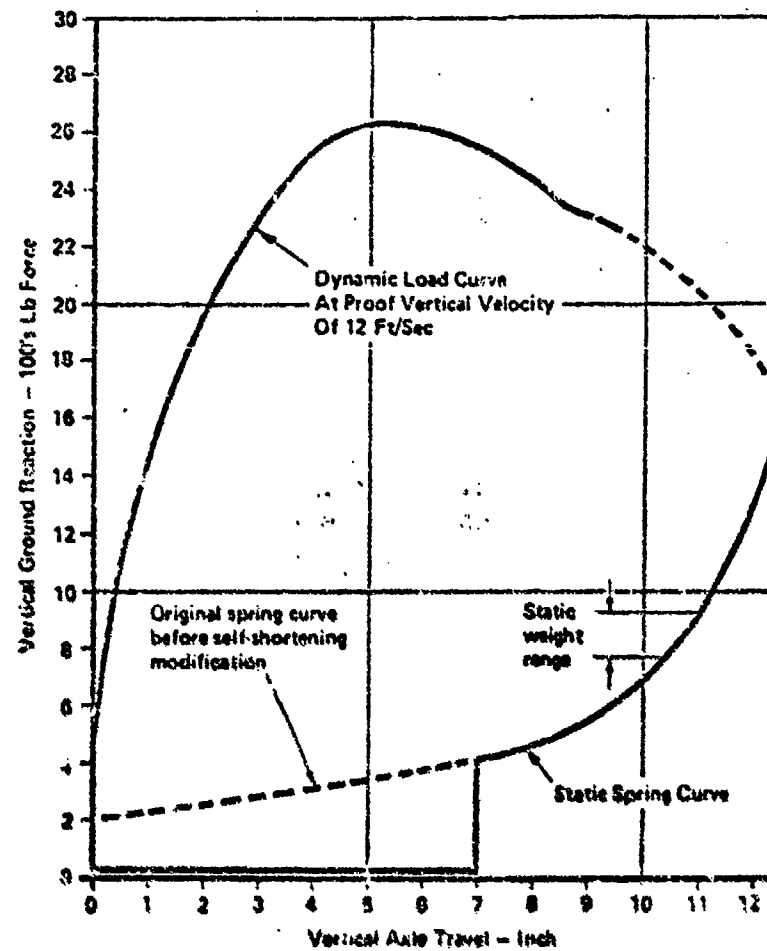
# Leading Particulars



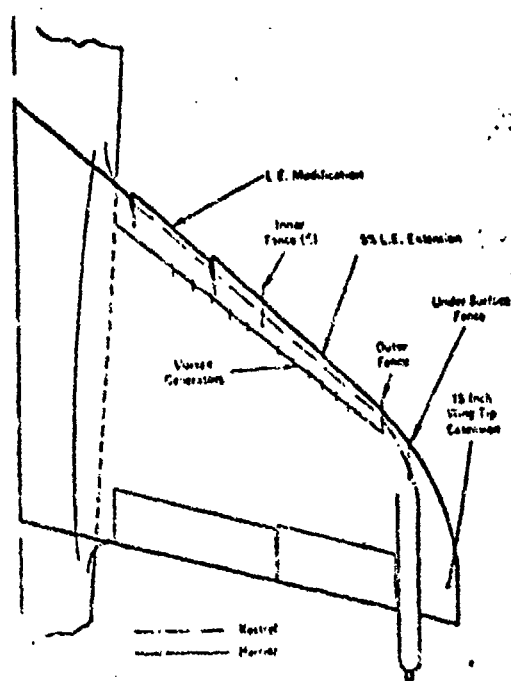




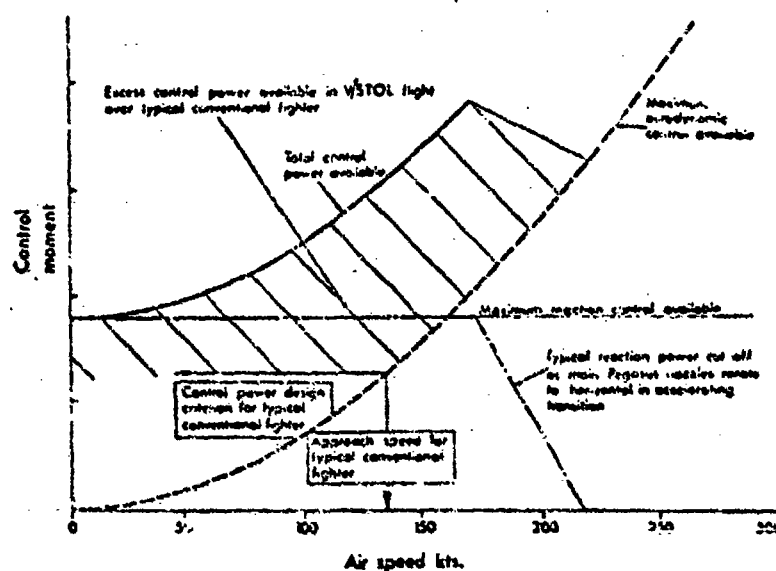
Main Leg Characteristics



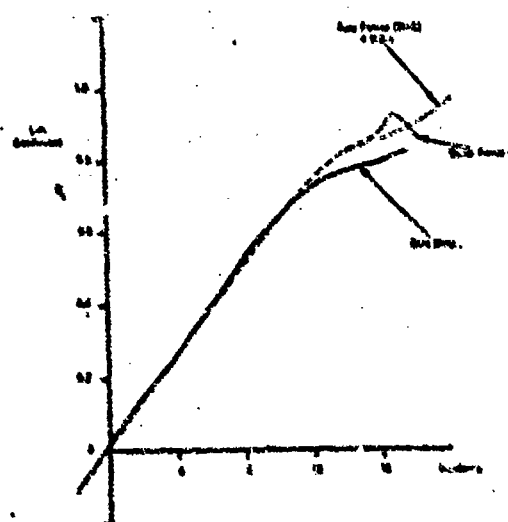
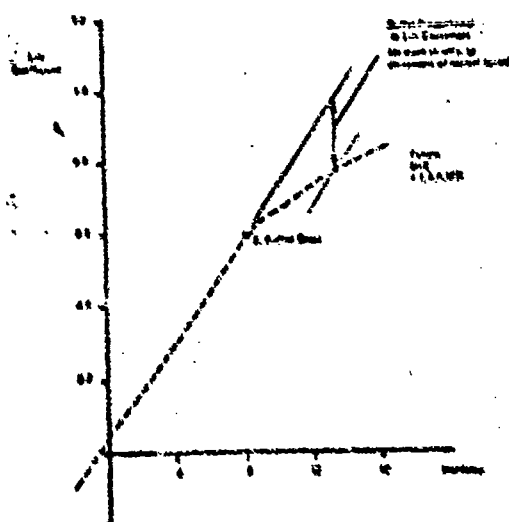
-The main undercarriage spring curve.



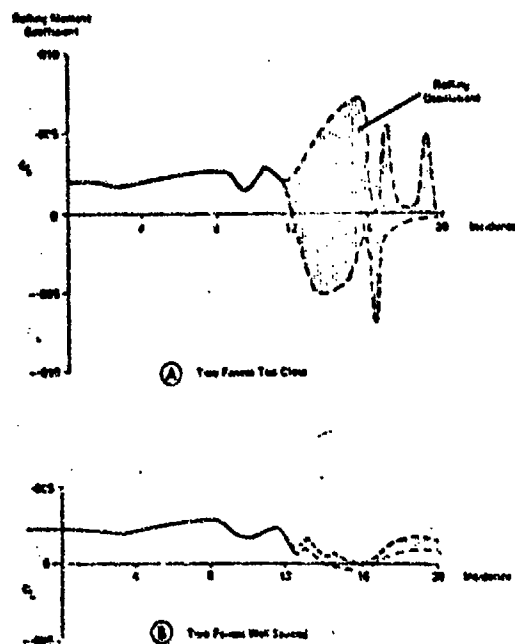
Planform of Harrier wing, showing Kestrel basis.



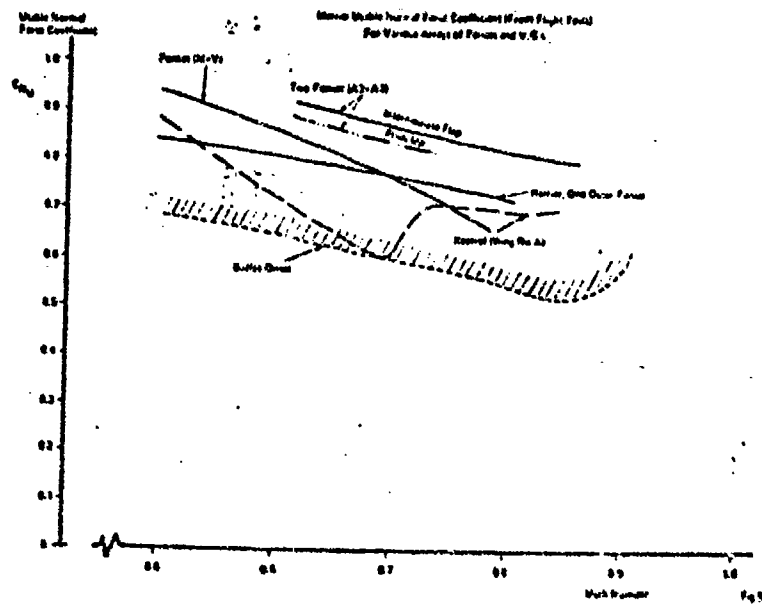
-Control power available in relation to airspeed.



Typical lift curves (a) buffet relationship, (b) effects of fences.

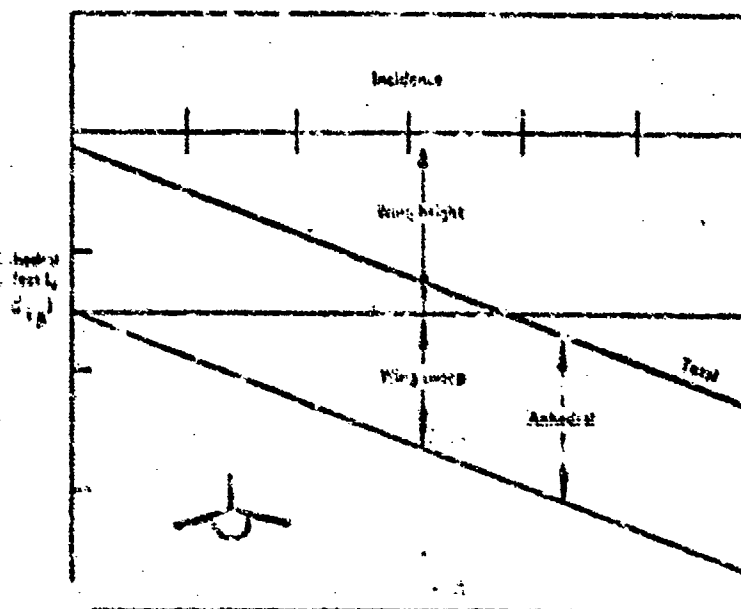


Roll unsteadiness with  
two fences:  
(a) too close  
(b) properly spaced.

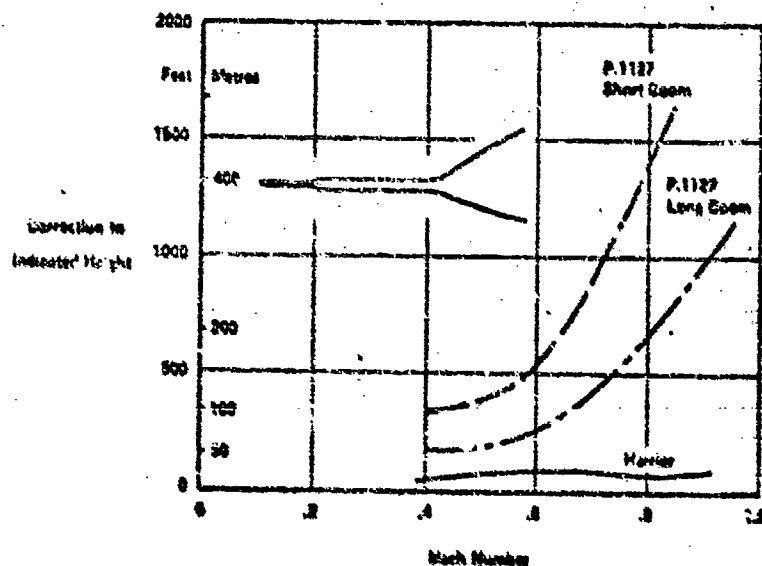


Typical usable normal-  
force coefficients:  
various fences and flaps  
on Harrier.

## STABILITY AND CONTROL



The various contributions needed to bring the total dihedral effect to an acceptable level.



The position errors achieved at low altitude with a short, light weight piezostatic probe compared with an uncompensated probe of the same length.